Boomerang 2003: Measuring the Polarization of
the
Cosmic Microwave Background Radiation

A Dissertation submitted in partial satisfaction
of the requirements for the degree of

Doctor of Philosophy

in

Physics

by

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February 2008
Boomerang 2003: Measuring the Polarization of the
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To Erin, for her sacrifices through all these long years.
Acknowledgements

Along the path of my graduate studies, there have been many people who have helped me in my endeavors. I would like to thank my advisor, John Ruhl, for giving me opportunities and support over the span of many years. Without his support this thesis would never have been completed.

My family has been a continuous source of support over the years, and I cannot thank them enough for their understanding. Without their help, this document would not exist.

Through the dark days of hardware testing and failures, Tom Montroy was an excellent colleague and mentor. His patience and knowledge will always be appreciated. His persistence in solving difficult problems was an inspiration to me.

I would like to thank the staff in the UCSB physics department. Mike Deal personally saved me an incredible amount of time over the course of our research at UCSB. Small words here can never measure the efficiency he brought to our daily work. In the past few months, Kerri O’Connor has gone to incredible lengths to enable me to complete the administrative steps necessary to defend.

The Boomerang experiment has only succeeded through the tireless work of people all over the world. My final thanks go to all of them: P. A. R. Ade, J. J. Bock, J. R. Bond, J. Borrill, A. Boscaleri, P. Cabella, C. R. Contaldi, B. P. Crill, P. de Bernardis, G. De Gasperis, A. de Oliveira-Costa, G. De Troia, G. Di Stefano, E. Hivon, A. H. Jaffe, A. E. Lange, C. J. MacTavish, S. Masi, P. D. Mauskopf, A. Melchiorri, T. E. Mon-
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Abstract

Boomerang 2003: Measuring the Polarization of the Cosmic Microwave Background Radiation

Theodore Schuyler Kisner

In early 2003, the Boomerang telescope flew for fifteen days over the Antarctic continent suspended from a balloon at an approximate altitude of 100,000 feet. Using a cryogenically cooled, bolometric receiver, it made measurements of the intensity and polarization of the Cosmic Microwave Background (CMB) Radiation in two overlapping sky patches of 100 and 800 square degrees. A spatial analysis of this data provides confirmation of previous measurements of the multipole angular power spectrum of the temperature anisotropies of the CMB. This data also provides power spectra of the polarization and temperature-polarization correlations that are competitive with previous experiments. Cosmological parameters estimated from these angular power spectra are consistent with a “standard” LCDM universe where inflation was adiabatic.
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Chapter 1

Introduction

Imagine that human eyes could “see” light whose wavelength was several thousand times longer than the colors of the visible spectrum. An astronaut gazing into space would see an almost perfectly uniform glow coming from all directions, and originating from beyond the furthest galaxies. The photons of this ancient afterglow have traveled for more than thirteen billion years to reach us. This Cosmic Microwave Background (CMB) radiation provides the earliest possible “snapshot” of the universe in its infancy. Detailed measurements of this light can allow us to constrain the composition and properties of the universe itself.

Our current best picture of the large scale universe and its history includes a fiery birth in an event known as the Big Bang, and a subsequent expansion and cooling phase which is still going on today. This “standard model” of cosmology has been built up gradually, beginning with Einstein’s theory of relativity. The observations of Edwin
Hubble and others in the early 1900’s showed that the recession velocity of galaxies increase linearly with increasing distance from our own [59]. This discovery that the universe was expanding challenged the contemporary ideas of a static universe. Armed with these observations, scientists began trying to build a picture of our universe and its history.

1.1 Isotropic and Homogeneous Spacetime

An early conjecture made by Einstein was that on the largest scales there should be no “special” places (universe is homogeneous) or directions (universe is isotropic) in space. Although motivated by Copernican ideals, later observations of distant galaxies and the CMB have been consistent with this hypothesis. If we start with Einstein’s equation from general relativity

\[ G_{\mu\nu} = 8\pi G T_{\mu\nu}, \]

(1.1)

and consider a homogeneous and isotropic universe, then we effectively force the left hand side to be a space of constant curvature[76]. Another consequence is that the stress-energy tensor on the right hand side takes the form of a perfect fluid. The spacetime metric in spherical coordinates is given by the Friedman-Lemaître-Robertson-Walker (FLRW) expression [76]:

\[ ds^2 = c^2 dt^2 + a(t)^2 \left[ \frac{dr^2}{1 - r\kappa} + r^2 d\theta^2 + r^2 \sin \theta d\phi^2 \right], \]

(1.2)
where $t$ is proper time and $\kappa$ is zero, positive, or negative depending on whether the curvature of the universe is flat, positive, or negative. The scale factor $a(t)$ parameterizes the expansion. The fact that we are dealing with a perfect fluid leaves us with just two components of Einstein’s equation. Following the treatment in [76], we are left with

$$3 \left( \frac{\dot{a}}{a} \right)^2 = 8\pi G \rho - \frac{3\kappa}{a^2} \quad \text{and} \quad \quad (1.3)$$

$$3 \left( \frac{\ddot{a}}{a} \right) = -4\pi G \left( \rho + \frac{3P}{c^2} \right), \quad (1.4)$$

where dots indicate differentiation with respect to proper time, $\rho$ is the energy density, and $P$ is the pressure. The Hubble parameter is defined to be

$$H = \frac{\dot{a}}{a}. \quad (1.5)$$

The dynamics of the universe and its curvature depend on the density and pressure (equation of state) of its components. Also note that from equation (1.4), if the universe has only “normal” matter and radiation obeying the equations of state in (1.7), then the acceleration of the scale factor is negative. This implies that the scale factor (and hence the universe) must either be expanding or contracting. Since early observations showed that the universe was expanding, the simplest conclusion was that at some point in the past, everything in the universe was located at a single infinitesimal point. The birth of the universe from this singularity was the Big Bang.

The conclusion that the universe could not be static in general relativity led to Einstein’s famous addition of a “cosmological constant” to his relativity equation in order to allow for a static universe. After the discovery that the universe really was expanding,
this constant was retracted. In the modern era, there is a large amount of evidence for the existence of some form of “dark energy” which mimics the effects of a cosmological constant. We usually consider this energy to be some kind of potential associated with spacetime itself. This additional energy density is simply included as another component of the total energy density.

We can gain further insight into the evolution of the universe by combining equations (1.3) and (1.4) to arrive at the following form of the continuity equation:

$$\dot{\rho} + 3H \left( \frac{\rho + P}{c^2} \right) = 0.$$  \hspace{1cm} (1.6)

The energy density of the components of the universe evolve differently depending on their equation of state:

- **Matter:** \( P = 0 \) \( \Rightarrow \rho \propto a^{-3} \)
- **Radiation:** \( P = \frac{\rho c^2}{3} \) \( \Rightarrow \rho \propto a^{-4} \)  \hspace{1cm} (1.7)
- **Cosmological Constant:** \( P = -\rho c^2 \) \( \Rightarrow \rho \) is constant

Now let’s examine the case of a flat universe (\( \kappa = 0 \)). Equation (1.3) now becomes

$$3H^2 = 8\pi G\rho.$$  \hspace{1cm} (1.8)

Solving for the density, we arrive at what is known as the “critical” density of the universe:

$$\rho_{\text{critical}} = \frac{3H^2}{8\pi G}.$$  \hspace{1cm} (1.9)

If the density is greater than this quantity, the universe will have positive curvature, and the expansion will eventually slow down and reverse itself. If the density is less than
critical, the universe has negative curvature, and the expansion will continue indefinitely. The “density parameter” is the ratio of the actual density to the critical density:

$$\Omega = \frac{\rho}{\rho_{\text{critical}}}.$$  \hspace{1cm} (1.10)

If we use the results of equation (1.7) and substitute them into equation (1.3), we can solve for how the scale factor changes with proper time[54]:

- **Matter:** \( a \propto t^{2/3} \)
- **Radiation:** \( a \propto t^{1/2} \)
- **Cosmological Constant:** \( a \propto \exp(\text{constant} \times t) \) \hspace{1cm} (1.11)

Of course, since the universe is made up of multiple components, the evolution of the total density will be some combination of the expressions in (1.7). The evolution of the scale factor will depend on which components dominate the density at a given time.

### 1.2 Nucleosynthesis

In the 1940’s, Alpher, Gamow and Herman worked on calculations which proposed that shortly after the Big Bang the high temperatures and densities in the universe would have led to nuclear fusion. They postulated that this could explain the relative abundances of the light elements (beryllium and lighter) that we observe today[2, 1, 18].

At about one second after the Big Bang, the temperature had dropped to around \(10^{10}\)K, and neutrons and protons were no longer in thermal equilibrium through weak
interactions. Their number densities were now fixed at a ratio of approximately 1 neutron to 10 protons. This ratio is set by the efficiency of the neutron → proton conversion as the temperature nears the equivalent mass difference between the two particles [16]. It is also affected by neutron decay prior to nucleosynthesis. A couple of minutes later the temperature had dropped to ≈10^9 K, and protons and neutrons could combine to form deuterium and eventually ^4\text{He}. Nearly all of the neutrons are quickly bound up in ^4\text{He} nuclei, with only a small residual number remaining in deuterons. Eventually, the universe expands and cools enough that these fusion reactions no longer occur.

This process of binding up the available neutrons into elements heavier than hydrogen is well determined by known nuclear reaction rates. The only free parameter in the process is the baryon to photon ratio- so one free parameter determines the primordial abundances of all the light elements. A generic prediction of BBN for a large range of baryon densities is that the mass fraction of ^4\text{He} to H should be ≈ 25%. This value agrees roughly with our observations, and is much higher than the amount predicted by stellar production alone. This fact is a key piece of evidence for the Big Bang model.

The mass fraction of primordial deuterium decreases rapidly at higher baryon density, since more of the deuterium is able to fuse into ^4\text{He}. Also, stellar processes do not produce appreciable amounts of deuterium, so nearly all of the deuterium in the universe was created during BBN.

In order to measure the baryon density, there are two separate approaches: we can try to directly measure the elemental abundances in “pristine” regions of the universe.
(metal-poor gas clouds, etc), or we can estimate the baryon density from observations of the CMB (see chapter 2). Figure 1.1 shows some recent CMB results for the baryon density from the B03 results combined with the WMAP satellite (see section 2.3). The agreement with observed abundances is remarkably close. The $^4$He results are within 1% of each other, and the deuterium results agree to within $<1\sigma$. The deuterium numbers are more robust, since deuterium is not created in appreciable quantities after BBN. The small discrepancy for the $^4$He results is likely due to observational errors in separating primordial material from that enriched by stellar production. There is obviously still some work to be done, and there may be additional sources of error in the $^7$Li observations[12].

1.3 The Cosmic Background Radiation

As noted in the early papers on BBN, the universe should have gone through a phase when the radiation and electrons were in thermal equilibrium. The radiation would be freed as neutral atoms formed. As the universe expanded, this radiation would have cooled significantly, but should still be present. Alpher and Herman even proposed several models in which the current radiation temperature should be either 1K or 5K[2]. This research did not receive much notice until it was refined and expanded upon by Robert Dicke, Jim Peebles, David Wilkinson in the 1960’s. These three set out to measure this background temperature, but were beaten to it by Arno Penzias and Robert Wilson. In 1965, Penzias and Wilson detected an unexplained background source at
Figure 1.1: This figure compares the B03 + WMAP measurements of the baryon density from the CMB with those from direct observations. The blue curves represent the calculated abundances as a function of baryon density. The red lines are the $1\sigma$ confidence limits on the baryon density from the CMB data. The green hashed regions are the primordial abundances of $^4$He, Deuterium, and $^7$Li obtained from observations. (Based on a figure from [12]). The 1% discrepancy in $^4$He and the discrepancy in $^7$Li are likely due to the difficulty of selecting and observing material that has not been enriched by stellar processes.
radio frequencies with a large feed antenna[60]. Dicke, Peebles, Roll, and Wilkinson explained this as the radiation left over from the early universe[14]. Since this time, the thermal spectrum of the CMB has been measured with great precision (see section 2.3).

The existence of the CMB is one of the strongest confirmations of the Big Bang model. None of the proposed “steady-state” models of the universe predicted the existence of the CMB. Up until about 300,000 years after the Big Bang ($z = 1100$), the universe was a hot plasma. Photons were coupled to free electrons via Thomson scattering. As the universe cooled, free electrons became increasingly bound to protons and were unavailable to scatter electrons. Since neutral Hydrogen gas is transparent, the photons streamed off in their direction of last scattering. At the time it was emitted, the mean blackbody temperature of this radiation was $\approx 3000\text{K}$. The expansion of the universe over the intervening 13 billion years has caused a redshift in the wavelength of this light, whose blackbody curve now peaks at $2.7\text{K}$ in the microwave part of the spectrum. The deviations of this light from a perfect blackbody are on the order of one part in $10^5$. Chapter 2 discusses the CMB in detail.

### 1.4 Inflation

Although the original Big Bang theory explained many of our observations about the universe, there were still some puzzles. One example concerning the CMB is that the blackbody temperature of the light is remarkably uniform in all directions across the sky. This indicates that at some time prior, the entire sky was in thermal equilib-
rium. However, equilibrium would require that all regions have time to interact (via
radiation). Given the light travel time between widely separated regions on the sky, this
thermalization would not have had time to occur during the age of the universe up to
the time of decoupling! This conundrum is known as the “horizon problem”.

Next consider the fact that our local universe appears relatively “flat”. In other
words, the average density of the universe seems very close to the critical density. In
order for our universe to still exist nearly 14 billion years later, the density just after the
Big Bang would have had to be equal to the critical density to within one part in $10^{15}$
[37]. If the density had deviated even slightly from this, the universe would long ago
have either recollapsed on itself or expanded so rapidly that galaxies would not have
formed. This observation is known as the “flatness problem”.

Since the speed of light in a vacuum is constant and we have no reason to believe
that the universe is required to have exactly the critical density, the above observations
are puzzling. The best current explanation for these is that the universe went through a
rapid period of inflation at about $10^{-35}$ seconds after the Big Bang [21]. Spacetime itself
expanded faster than the speed of light, and our entire observable universe expanded
from a much smaller region which was in thermal equilibrium prior to this inflation.
The rapid expansion of a small patch of the universe means that any curvature has been
effectively “flattened out” (since any sufficiently small piece of the universe appears
roughly flat). Another way to say this is that the radius of curvature of the universe has
been inflated to some value much larger than our observable portion of the universe.
In addition to solving the horizon and flatness problems, inflation also gives us an explanation for the spectrum of initial density perturbations that led to anisotropies in the CMB. Quantum fluctuations just after the Big Bang were blown up by inflation to scales larger than the horizon size [35]. After inflation, these fluctuations remained as perturbations in the background metric. As the horizon gradually expands after inflation, larger modes can interact. The modes enter the horizon with the same amplitude as when they were “frozen” in place by inflation pushing them outside the horizon. A generic prediction of inflationary models with exponential expansion is that the spectrum of resulting perturbations is nearly scale-invariant. This means that as larger k-modes enter the horizon after inflation, the power in each mode is nearly equal.
Chapter 2

The CMB

At a redshift of $\approx 1100$, the universe had cooled sufficiently for free electrons to combine with available nuclei and form neutral atoms. The photons of the CMB were then “decoupled” from the electrons, and could finally stream freely through space. In order to draw any kind of conclusion from observations of the CMB, we need to consider what properties of the universe influence the nature of the light emitted at last scattering. We also need to consider what factors may affect these photons as they travel across the breadth of the observable universe to our vantage point.

2.1 Spatial Analysis of Polarized Light

Before discussing the physical processes that give rise to CMB photons and how these are measured today, it is necessary to establish a framework for describing polarized light. In CMB analysis, we want to be able make maps of the sky, as well as
compress this map information into angular power spectra. See section 5.10 for details on extracting cosmological information from the power spectra.

2.1.1 Stokes Parameters

The electric field of an incoming electromagnetic wave of frequency \( \nu \) in the plane perpendicular to its travel is \([29, 34]\)

\[
\vec{E}(\nu, t) = E_x(t) \cos(2\pi \nu t + \phi_x) \hat{x} + E_y(t) \cos(2\pi \nu t + \phi_y) \hat{y}.
\] (2.1)

The intensity and polarization can be described by the Stokes parameters \([29]\). In the case of our experiment, we are measuring light of many frequencies within a particular band rather than monochromatic light. This is equivalent to time-averaging the Stokes parameters over some period much longer than the period of one oscillation:

\[
\begin{align*}
I &= \langle E_x^2(t) + E_y^2(t) \rangle \\
Q &= \langle E_x^2(t) - E_y^2(t) \rangle \\
U &= 2 \langle E_x(t) E_y(t) \cos(\phi_x - \phi_y) \rangle \\
V &= 2 \langle E_x(t) E_y(t) \sin(\phi_x - \phi_y) \rangle
\end{align*}
\] (2.2)

For the CMB, the Thomson scattering which produces the polarization is only capable of inducing a linear polarization. For this reason we usually ignore the \( V \) Stokes parameter, which describes the amount of circular polarization. It is also worth noting that the intensity, \( I \) is independent of the choice of coordinate system, while \( Q \) and \( U \) are not. The \( Q \) Stokes parameter measures the difference in intensity between the \( x \)
and $y$ components, while the $U$ Stokes parameter is the intensity difference between the $-45^\circ$ and $45^\circ$ directions.

### 2.1.2 Power Spectra

The angular power spectra of the intensity and polarization of the CMB contain the same cosmological information as the sky maps (assuming the radiation field is nearly isotropic and generated by Gaussian random processes), while greatly reducing the number of data points needed for a likelihood analysis. Computing a power spectrum requires that we expand the sky data in terms of spherical harmonics and then calculate correlations between the expansion coefficients. This process is straightforward for the intensity, which is a scalar quantity on the sphere. For polarization, the Stokes parameters are problematic, because we cannot define a rotationally invariant, orthonormal basis on the sphere. Following the treatment in [79], we instead construct two spin-2 combinations of the Stokes parameters which transform under rotation about the axis of travel as

\begin{align}
\left[ Q + iU \right]'(\theta, \phi) &= e^{-2i\psi} \left[ Q + iU \right](\theta, \phi) \quad \text{and} \\
\left[ Q - iU \right]'(\theta, \phi) &= e^{2i\psi} \left[ Q - iU \right](\theta, \phi). \quad (2.3)
\end{align}
So now we can expand the temperature and polarization in terms of spin zero and spin two spherical harmonics respectively:

\[ T(\theta, \phi) = \sum_{\ell m} a_{T,\ell m} Y_{\ell m}(\theta, \phi) \]  
(2.5)

\[ [Q + iU](\theta, \phi) = \sum_{\ell m} a_{2,\ell m} 2Y_{\ell m}(\theta, \phi) \]  
(2.6)

\[ [Q - iU](\theta, \phi) = \sum_{\ell m} a_{-2,\ell m} -2Y_{\ell m}(\theta, \phi). \]  
(2.7)

Using the orthogonality of the spherical harmonics, we can express the expansion coefficients as:

\[ a_{T,\ell m} = \int \sin(\theta)d\theta d\phi Y_{\ell m}^*(\theta, \phi) T(\theta, \phi) \]  
(2.8)

\[ a_{2,\ell m} = \int \sin(\theta)d\theta d\phi 2Y_{\ell m}^*(\theta, \phi) [Q + iU](\theta, \phi) \]  
(2.9)

\[ a_{-2,\ell m} = \int \sin(\theta)d\theta d\phi -2Y_{\ell m}^*(\theta, \phi) [Q - iU](\theta, \phi). \]  
(2.10)

Ideally, we want to deal only with rotationally invariant quantities. To do this, we can make use of the spin raising and lowering operators for spin weighted functions discussed in the appendix of [79], and based on the earlier work of Goldberg [19] and Newman and Penrose [57]. In spherical coordinates, these operators are given by:

\[ \vec{\partial} = \left[ \sin^s(\theta) \left( \frac{\partial}{\partial \theta} - \frac{i}{\sin(\theta)} \frac{\partial}{\partial \phi} \right) \sin^{-s}(\theta) \right] \]  
(Raising)  
(2.11)

\[ \vec{\partial} = \left[ \sin^{-s}(\theta) \left( \frac{\partial}{\partial \theta} + \frac{i}{\sin(\theta)} \frac{\partial}{\partial \phi} \right) \sin^s(\theta) \right] \]  
(Lowering).  
(2.12)

The derivative operators here are covariant derivatives, and the value at each point on the sphere depends on the surrounding quantities. If we use these operators \( s \) times on the spin-zero spherical harmonics, we can obtain expressions for the spin-\( s \) harmonics.
in terms of the spin-zero ones:

\[
sY_{\ell m}(\theta, \phi) = \left[ \frac{(\ell - s)!}{(\ell + s)!} \right]^{\frac{1}{2}} \bar{\Phi}^s Y_{\ell m}(\theta, \phi) \quad \text{for } s > 0 \tag{2.13}
\]

\[
sY_{\ell m}(\theta, \phi) = \left[ \frac{(\ell + s)!}{(\ell - s)!} \right]^{\frac{1}{2}} (-1)^s \bar{\Phi}^{-s} Y_{\ell m}(\theta, \phi) \quad \text{for } s < 0. \tag{2.14}
\]

If we use the raising and lowering operators twice on equations (2.6) and (2.7), we now have three spin-0 quantities that describe the temperature and polarization of the CMB:

\[
T(\theta, \phi) = \sum_{\ell m} a_{T, \ell m} Y_{\ell m}(\theta, \phi) \tag{2.15}
\]

\[
\bar{\Phi}^2 [Q + iU](\theta, \phi) = \sum_{\ell m} \left[ \frac{(\ell + 2)!}{(\ell - 2)!} \right] a_{2, \ell m} Y_{\ell m}(\theta, \phi) \tag{2.16}
\]

\[
\bar{\Phi}^2 [Q - iU](\theta, \phi) = \sum_{\ell m} \left[ \frac{(\ell - 2)!}{(\ell + 2)!} \right] a_{-2, \ell m} Y_{\ell m}(\theta, \phi). \tag{2.17}
\]

The expansion coefficients can now be written as:

\[
a_{T, \ell m} = \int \sin(\theta) d\theta d\phi \ Y_{\ell m}^*(\theta, \phi) \ T(\theta, \phi) \tag{2.18}
\]

\[
a_{2, \ell m} = \left[ \frac{(\ell + 2)!}{(\ell - 2)!} \right] \int \sin(\theta) d\theta d\phi \ Y_{\ell m}^*(\theta, \phi) \ \bar{\Phi}^2 [Q + iU](\theta, \phi) \tag{2.19}
\]

\[
a_{-2, \ell m} = \left[ \frac{(\ell - 2)!}{(\ell + 2)!} \right] \int \sin(\theta) d\theta d\phi \ Y_{\ell m}^*(\theta, \phi) \ \bar{\Phi}^2 [Q - iU](\theta, \phi). \tag{2.20}
\]

In the study of CMB polarization, we usually define the scalar and pseudo-scalar fields called “E” and “B” (in analogy with electromagnetism) which are linear combinations of (2.16) and (2.17). E has even parity and B has odd parity. These definitions are physically motivated, since density perturbations can only generate E-type polarization. B-type signals are due to primordial gravity waves, lensing, or foreground
contamination [11]. These fields are given by:

\[
E(\theta, \phi) = -\frac{1}{2} \left( \bar{\delta}^2 [Q + iU] (\theta, \phi) + \bar{\delta}^2 [Q - iU] (\theta, \phi) \right) \tag{2.21}
\]

\[
B(\theta, \phi) = \frac{i}{2} \left( \bar{\delta}^2 [Q + iU] (\theta, \phi) - \bar{\delta}^2 [Q - iU] (\theta, \phi) \right). \tag{2.22}
\]

The \( E \) and \( B \) expansion coefficients are then:

\[
a_{E,\ell m} = -\frac{a_{2,\ell m} + a_{-2,\ell m}}{2} \tag{2.23}
\]

\[
a_{B,\ell m} = \frac{i (a_{2,\ell m} - a_{-2,\ell m})}{2}. \tag{2.24}
\]

\( E \) and \( B \) are non-local, and depend on the global properties of the polarization. Measuring the angular power spectra of the CMB temperature and polarization is then reduced to measuring all possible correlations between the \( T \), \( E \), and \( B \) expansion coefficients. These power spectra are [79]:

\[
C_{TT,\ell} = \frac{1}{2\ell + 1} \sum_m \langle \hat{a}_{T,\ell m}^* \hat{a}_{T,\ell m} \rangle \tag{2.25}
\]

\[
C_{EE,\ell} = \frac{1}{2\ell + 1} \sum_m \langle \hat{a}_{E,\ell m}^* \hat{a}_{E,\ell m} \rangle \tag{2.26}
\]

\[
C_{BB,\ell} = \frac{1}{2\ell + 1} \sum_m \langle \hat{a}_{B,\ell m}^* \hat{a}_{B,\ell m} \rangle \tag{2.27}
\]

\[
C_{TE,\ell} = \frac{1}{2\ell + 1} \sum_m \langle \hat{a}_{T,\ell m}^* \hat{a}_{E,\ell m} \rangle \tag{2.28}
\]

\[
C_{TB,\ell} = \frac{1}{2\ell + 1} \sum_m \langle \hat{a}_{T,\ell m}^* \hat{a}_{B,\ell m} \rangle \tag{2.29}
\]

\[
C_{EB,\ell} = \frac{1}{2\ell + 1} \sum_m \langle \hat{a}_{E,\ell m}^* \hat{a}_{B,\ell m} \rangle. \tag{2.30}
\]

All of the correlations between \( B \) and \( T \) or \( E \) should be zero, since \( B \) has a different parity. The shapes of the observed angular power spectra depend critically on the
processes occurring in the universe at the time that the CMB decoupled from matter.

Section 5.10 describes how we can use a Bayesian analysis to give us an estimate of the most likely cosmological parameters that would have given rise to our measured power spectra.

### 2.2 Physical Causes of Anisotropy

In the previous chapter, we saw how the Einstein and continuity equations for the large scale isotropic universe can be used to determine the evolution of the average density of various components. Now we consider small perturbations around this solution and their imprint in the CMB at the \(10^{-5}\) level. The amplitude and spatial distribution of temperature and polarization fluctuations in the CMB can tell us a great deal about the physical processes and environment at the time of decoupling. To make the connection between our measurements and the physics involved, we must have a model that relates various cosmological parameters to the spatial anisotropies that we see projected on the sky.

#### 2.2.1 Acoustic Oscillations Before Decoupling

The universe before decoupling was a hot, opaque plasma. Baryons and electrons interacted via coulomb forces, and electrons interacted with photons via Thomson scattering. Because of this, it is convenient to think of the pre-decoupling universe as a tightly coupled photon-baryon fluid which interacts gravitationally with the ambient
dark matter distribution. The state of this fluid depends on the details of its composition and on its initial conditions. A full, relativistic treatment of the dynamics of perturbations in this fluid must include the evolution of the density and equation of state of all components. See for example the treatment in [46].

Without going into this level of detail, we can still gain significant insight into the response to small density perturbations in this fluid [28]. In this case, we will make the simplifying assumptions of a flat universe and that the density perturbations have no shear or vorticity (i.e. the “perfect fluid” assumption). We also assume that the perturbations are small enough that linear perturbation theory applies. In hindsight, we know that this is valid due to the small amplitude of the CMB anisotropies compared to the overall temperature signal. These assumptions explicitly ignore possible tensor perturbations caused by primordial gravity waves.

In this useful toy model, our choice of Newtonian (longitudinal) gauge and the other assumptions above have reduced the free parameters to the Newtonian potential ($\phi$) and the gravitational redshift ($\psi$) [54]. These quantities are equal for a perfect fluid. The Einstein equations for a density perturbation of $\rho + \delta \rho$ and corresponding pressure perturbation $P + \delta P$ are then reduced to (derivatives are with respect to conformal time) [46]:

\[
\ddot{\phi} + H \left(2\dot{\phi} + \dot{\psi}\right) - \left(2\dot{H} + H^2\right)\psi - \frac{1}{3} \nabla^2 (\phi - \psi) = \frac{4\pi G a^2}{3} \delta P \\
-\nabla^2 \phi + 3H \left(\dot{\phi} + H\dot{\psi}\right) = -4\pi G a^2 \delta \rho \\
-\nabla^2 \left(\dot{\phi} + H\dot{\psi}\right) = 4i\pi G a^2 (\rho + P) \theta.
\]
Where we have defined a quantity, $\theta$, which represents the divergence of the fluid velocity. All of the potentials and perturbations in the above equation can be expanded in three dimensional Fourier modes parameterized by the wavevector, $\vec{k}$. Since we are dealing with linear perturbation theory, we can treat each mode independently. In this Fourier expansion, for a given $k$ mode, the Laplacian operator gives us a factor of $-k^2$.

To complete this picture, we need to use the continuity equations (equations of motion) for the photons and baryons. In CMB research, we usually deal with temperature fluctuations, rather than photon density. The Stefan-Boltzmann law relates the two quantities:

$$\rho_\gamma = \sigma T^4 \rightarrow \delta_\gamma = \frac{\Delta \rho_\gamma}{\rho_\gamma} = \frac{4\sigma T^3 \Delta T}{\sigma T^4} = \frac{4\Delta T}{T} = 4\delta_T. \quad (2.34)$$

The equations (in Fourier space) for the photons and the baryons with Thomson scattering then become [54]:

$$\dot{\delta}_T = -\frac{1}{3} \theta_T - \dot{\phi} \quad (2.35)$$
$$\dot{\theta}_T = k^2 \delta_T + k^2 \psi + an_e \sigma_T (\theta_b - \theta_\gamma) \quad (2.36)$$
$$\dot{\delta}_b = -\theta_b + 3\dot{\phi} \quad (2.37)$$
$$\dot{\theta}_b = -H \theta_b + c_s^2 k^2 \delta_b + k^2 \psi^2 + \frac{4\rho_\gamma}{3\rho_b} an_e \sigma_T (\theta_\gamma - \theta_b). \quad (2.38)$$

Where $c_s$ is the sound speed in the fluid. Because of the tight coupling between these components, we know that their temperatures and fluid velocities are equal. Using this information, we can combine the continuity equations for the two components to arrive
at:
\[
\dddot{\delta_T} + \frac{R}{1 + R} H \dot{\delta_T} + c_s^2 k^2 \delta_T = -\ddot{\psi} - \frac{R}{1 + R} H \dot{\phi} - \frac{k^2}{3} \psi.
\]
(2.39)

where \( R \) represents the baryon to photon ratio \( (R = 3\rho_b/4\rho_\gamma) \), \( k \) is the mode in the Fourier expansion, and the sound speed is defined as
\[
c_s^2 = \frac{1}{3(1 + R)}.
\]
(2.40)

This is the equation of a damped (by the expansion) and driven (by metric perturbations) harmonic oscillator. The potential wells set up by the metric perturbations cause the photon-baryon fluid to stream into them. As the fluid falls in it is compressed and heated, and photon pressure provides a restoring force and leaves to rarefaction. As the sound horizon expands with the universe, larger modes are able to oscillate. When the photons dominate the fluid \( (R << 1) \), these oscillations have the form:
\[
[\delta_T + \psi] (\eta_*) = \frac{1}{3} \psi \cos (k c_s [\eta_* - \eta_{\text{crossing}}]) \quad \eta_* = \text{time at decoupling}.
\]
(2.41)

In other words, the effective temperature of one k-mode in the oscillating fluid at decoupling depends on the size of the initial perturbation, on how much time has elapsed since that mode entered the sound horizon, and on the gravitational redshift from climbing out of the potential wells. At decoupling, modes larger that the sound horizon have not had a chance to oscillate. The effective temperature of these large scale modes is due only to the gravitational redshift of photons climbing out the potential wells. This is known as the Sachs-Wolfe effect [68], and means that large-scale overdense regions appear as relatively cold regions in our observations. As the universe expands and nears
the time of decoupling, the baryon density can no longer be ignored [27]. The baryons shift the zero point of the oscillations and enhance the compression phase. At small angular scales, the wavelengths of the acoustic oscillations approach the mean free path for scattering. This causes photons and electrons to diffuse past each other instead of scattering, and leads to damping of small scale oscillations.

2.2.2 Generating Polarization

We have seen that Thomson scattering is an important process in the plasma state of the universe before decoupling. When a beam of incoming photons scatters off electrons, the outgoing photons in a given direction are linearly polarized with an intensity equal to the parallel component of the incoming photon. If the intensity of incident photons is uniform from all directions, then there is no net polarization caused by this process. If the distribution of photon intensities has a non-zero quadrupole moment, then there will be a net linear polarization of the outgoing photons.

We can show this explicitly by starting with the differential cross section for Thomson scattering[29],

\[ \frac{d\sigma}{d\Omega} = \frac{3\sigma_T}{8\pi} |\hat{e}' \cdot \hat{e}|^2 \]

where \( \sigma_T = \frac{8\pi}{3} \left( \frac{e^2}{mc^2} \right)^2 \),

and then computing an expression for the Stokes parameters of the outgoing photons. In the above expression, \( \hat{e} \) is the polarization unit vector of the incoming plane wave, and \( \hat{e}' \) is the outgoing unit vector. Following the treatment in [38], we consider a beam of unpolarized photons scattering off of an electron into the \( \hat{z}' \) direction (see figure 2.1).
If we define the incoming intensity to be $I$, and measure the outgoing Stokes parameters with respect to the $\hat{x}'$ direction, then these outgoing Stokes parameters are:

$$I' = \frac{3\sigma_T}{8\pi} I \left(1 + \cos^2(\theta)\right)$$  \hspace{1cm} (2.43)

$$Q' = \frac{3\sigma_T}{8\pi} I \sin^2(\theta)$$  \hspace{1cm} (2.44)

$$U' = 0.$$  \hspace{1cm} (2.45)

The $V$ Stokes parameter is zero, since the symmetry of Thomson scattering prevents the generation of circular polarization. The above expressions are for a single direction of incoming light. Now we need to integrate this over all possible incoming directions.

**Figure 2.1:** This diagram depicts photons scattering off an electron into the $\hat{z}'$ direction. The component of intensity perpendicular to the scattering plane is preserved, while the parallel component is attenuated based on the scattering angle.
and get an expression for the total outgoing Stokes parameters in the \( \hat{z}' \) direction. We need to remember to rotate the incoming coordinate systems about the \( \hat{z} \) axis so that the outgoing axes are aligned. Recall from equation (2.4) the transformation of the Stokes parameters under rotation. Using this fact, the integral over incoming directions gives us:

\[
I'(\hat{z}') = \frac{3\sigma_T}{16\pi} \int d\Omega \ (1 + \cos^2(\theta)) I(\theta, \phi) \tag{2.46}
\]

\[
Q'(\hat{z}') - iU'(\hat{z}') = \frac{3\sigma_T}{16\pi} \int d\Omega \ \sin^2(\theta) e^{2i\phi} I(\theta, \phi). \tag{2.47}
\]

Now we expand the input intensity field in terms of spherical harmonics,

\[
I(\theta, \phi) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta, \phi), \tag{2.48}
\]

and also rewrite the angular terms in the integral as sums of spherical harmonics:

\[
(1 + \cos^2(\theta)) = \frac{8\sqrt{\pi}}{3} Y_{00} + \frac{4}{3} \sqrt{\frac{\pi}{5}} Y_{20} \tag{2.49}
\]

\[
\sin^2(\theta) e^{2i\phi} = 4 \sqrt{\frac{2\pi}{15}} Y_{22}. \tag{2.50}
\]

Substituting these expressions into equations (2.46) and (2.47), we get

\[
I'(\hat{z}') = \frac{3\sigma_T}{16\pi} \int d\Omega \left[ \frac{8\sqrt{\pi}}{3} Y_{00} + \frac{4}{3} \sqrt{\frac{\pi}{5}} Y_{20} \right] \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta, \phi) \tag{2.51}
\]

\[
Q'(\hat{z}') - iU'(\hat{z}') = \frac{3\sigma_T}{16\pi} \int d\Omega \left[ 4 \sqrt{\frac{2\pi}{15}} Y_{22} \right] \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta, \phi). \tag{2.52}
\]
Because of the orthogonality of the spherical harmonics, only some multipoles are selected from the sum by the integration. The final solution is:

\[
I'(\hat{z}') = \frac{3\sigma_T}{16\pi} \left[ \frac{8\sqrt{\pi}}{3} a_{00} + \frac{4}{3} \sqrt{\frac{\pi}{5}} a_{20} \right] 
\]

\[
Q'(\hat{z}') - iU'(\hat{z}') = \frac{3\sigma_T}{16\pi} \left[ 4 \sqrt{\frac{2\pi}{15}} a_{22} \right].
\]

So at the time of decoupling, Thomson scattering can produce a net polarization if the photon intensity has a quadrapole moment in the electron’s rest frame. How can such a quadrapole be generated? Consider an electron just prior to last scattering which is streaming into an overdense region (see figure 2.2). The photon-baryon fluid “in front”

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.2.png}
\caption{As electrons stream into overdense regions around the time of decoupling, the photon-baryon fluid in front and behind the electrons appears to be moving away. Photons scattering from these directions are red shifted and so have a reduced intensity. Photons scattering from directions perpendicular to the electron’s motion are blue shifted.}
\end{figure}

of the electron has a higher velocity, since it has accelerated further into the potential well. The fluid “behind” the electron has a slower velocity. So in the electron’s rest
frame, the photon-baryon fluid along its axis of travel appears to be moving away, and is red shifted. The co-moving fluid perpendicular to the electron’s motion appears blue shifted, since all of the material is being compressed as it falls deeper into the potential well. This Doppler shifting sets up an effective quadrupole in the intensity of the photons scattering off streaming electrons.

It is important to note that this quadrupole can only be established at the end of the decoupling era. If the photons and baryons are too tightly coupled, then the electrons will only experience a monopole (temperature) and a dipole (due to its velocity) in the incoming radiation field. Also, although Thomson scattering can serve to create linear polarization as described above, this polarization is destroyed (and regenerated) upon subsequent scatterings. Because of this, observations of CMB polarization probe only the last scattering surface.

2.3 Previous Experiments

Now that we have considered the mechanisms which generate temperature and polarization fluctuations in the CMB at the time of last scattering, we can consider how these photons appear to us today. To an electron in the pre-decoupling plasma, the photon distribution has monopole, dipole, and (at decoupling) quadrupole moments. This describes the temperature fluctuations in space at $z = 1100$, but how does this correspond to observations today?
From our vantage point, we see the photon distribution projected onto the last scattering surface. The previously discussed acoustic oscillations are frozen in at decoupling. The largest possible k-mode which can reach compression at decoupling is the mode whose size equals the sound horizon at recombination. Other k-modes will be in various states of compression and rarefaction when decoupling happens. K-modes whose size is $1/2, 1/3, 1/4$, etc of the largest mode will be in alternating states of maximal expansion or compression. In real space at decoupling, regions of compression are relatively hot and regions of rarefaction are relatively cold. The light from real-space hot and cold regions gets projected along our line of sight, and then must climb out of any local potential wells (as discussed in section 2.2.1).

The physical anisotropies at decoupling correspond to angular features that we see today. They depend on the spectrum of k-space temperature fluctuations in the photon-baryon fluid at decoupling, the projection of the hot and cold regions onto the last scattering surface, the distance to this surface, the distribution of dark matter potential wells, and on the curvature of the intervening space. In flat space, a projected spatial inhomogeneity on the last scattering surface of size $\lambda$ will appear roughly as an angular feature of size[27],

$$\Theta = \frac{H\lambda}{c} \frac{(1 + z_{\text{CMB}})^2}{2 [z_{\text{CMB}} + 1 - \sqrt{z_{\text{CMB}} + 1}]}$$

(2.55)

which depends on the Hubble constant and the redshift of decoupling. At decoupling, the k-space modes which are at extrema of their motion correspond to hot and cold spots of
will give rise to peaks in the angular temperature power spectrum on the sky. The spectral peaks caused by progressively smaller oscillations form a harmonic series, with the multipole $\ell$-space location of these peaks given by [27],

$$\ell_n \approx \frac{n\pi D_s}{s_*},$$

(2.56)

where $D_s$ is the co-moving distance to the last scattering surface, and $s_*$ is the sound horizon at decoupling. In addition to a series of peaks, there are other spectral features that are affected by cosmological parameters. One effect is that the baryons shift the equilibrium point of the oscillations of the effective temperature. This leads to an accentuation of the compressional peaks (the first, third, etc). Another effect is that the diffusion damping attenuates peaks at higher $\ell$-values. At large angular scales (super-horizon), the angular power spectrum should be approximately flat if the initial perturbation spectrum is scale invariant.

Since the first CMB measurement by Penzias and Wilson, dozens of experiments have attempted to measure the angular power spectra of the CMB. Initial experiments were concerned simply with measuring the absolute temperature (monopole). The COBE satellite, launched in 1989, was the first experiment to detect anisotropies in the microwave background [4] and also measured the monopole temperature to an accuracy of 2mK [51]. The large scale anisotropies detected by COBE were super-horizon. After this initial detection, many experiments raced to see if there was a peak in the angular power spectrum due to the largest scale acoustic oscillations. Figure 2.3 shows the state of temperature anisotropy measurements prior to B03. Included in this plot
are results from the 1998 flight of BOOMERANG [67], as well as the first year results from the WMAP satellite [25] and several other experiments with high signal-to-noise detections of the anisotropies. As the temperature power spectrum was measured with

\[ \frac{(l+1)C_l}{2\pi} \]

0 500 1000 1500 2000 2500

0 2000 4000 6000 8000

Best Fit LCDM
MAXIMA-1
ACBAR
WMAP 1 Year
VSA
DASI
CBI
BOOMERanG 1998

Figure 2.3: This figure shows roughly the state of measurements of the CMB temperature anisotropies prior to the 2003 flight of BOOMERANG. The black curve is a best fit LCDM model. Data used in plot taken from [41], [25], [40], [65], [39], [15], and [67].

increasing accuracy, another generation of experiments (including B03) was readied to search for the polarization of the CMB. Figure 2.4 shows the status of these efforts prior to B03. Since the publication of the most recent results from BOOMERANG, other experiments have also published results consistent with a standard LCDM model. See section 5.11. From the angular power spectra, we can compute the most likely values
Figure 2.4: This figure shows the measurements of the CMB polarization anisotropies prior to the 2003 flight of BOOMERANG. Data used from [39](DASI) and [36](WMAP).

of various cosmological parameters that would have given rise to the projected acoustic oscillations. Section 5.10 covers this process in more detail.
Chapter 3

Design of the Boomerang Telescope

The Boomerang telescope was designed to be carried aloft by a high altitude balloon as part of NASA’s Long Duration Balloon (LDB) program. These balloons are launched from McMurdo Station, Antarctica, and rise to an altitude of 120,000 feet. The polar vortex winds during the austral summer generally carry the balloon on a circumpolar track lasting between 7 and 20 days. The dry, near-vacuum (3 Torr) conditions at the float altitude are excellent for observing the CMB.

The requirements of the LDB flight put constraints on the size and weight of our telescope, and also require that our telescope can survive the conditions at the float altitude of the balloon (low pressure, freezing temperatures, UV exposure, high cosmic ray flux, etc). Additionally, our science goals require that we have high enough sensitivity to measure CMB polarization with a reasonable signal to noise, and with good assur-
ance that our data is not contaminated by galactic foregrounds or systematic effects. These basic requirements lead us to many secondary design criteria.

3.1 The Balloon Gondola

The outer frame of the telescope is known as the balloon “gondola” (see figure 3.1). This structure is made of four inch square aluminum box beams and provides a support for all other telescope components. The gondola is attached to the balloon via a pivot mount which swivels and allows the telescope to freely rotate with respect to the balloon. The entire gondola is designed to scan in azimuth. By scanning the telescope, we ensure that the changing sky signal from the CMB occurs at optimal frequencies for our detectors (see section 3.6). The actual scanning is accomplished by using a motor to spin a weighted flywheel, which causes the telescope to rotate in order to conserve angular momentum[10]. The solar panels on the rear of the gondola provide up to 5kW of power at 28 volts DC. A set of standard lead acid car batteries provides a buffer against fluctuations in the power output of the panels. The exterior surface of the gondola is covered in standard housing insulation foam which is in turn covered with Mylar which has been aluminized on one side. The Mylar side is placed outwards. Optical light from the sun is reflected from the interior aluminized surface, and the high emissivity of the Mylar in the infrared helps keep the interior of the gondola cool. Even with extensive shielding against solar heating, some components with high power outputs (the data acquisition system and flight data recorder) were painted white to
further improve their emissivity. In addition to the shielding surrounding the gondola, two large “wings” and a “scoop” are mounted on the front of the telescope to reduce in-band contamination from the ground and the sun. With this shielding in place, the telescope is able make $120^\circ$ peak to peak scans in the anti-sun direction without any pick-up from solar radiation.

3.2 The Pointing Subsystem

Because our detector beams are roughly ten arcminutes across, it is important that we know our absolute pointing to within several arcminutes. To accomplish this, we use a complementary set of pointing sensors which include a differential GPS array, fixed...
and pointed sun sensors, a star tracking camera, gyroscopes, and an elevation sensor. The output of these sensors are then combined to form a complete pointing solution for the entire flight (see section 5.2).

The TANS-VECTOR[78] differential GPS array provides absolute pointing of the telescope at coarse resolution and with roughly 6’ drifts on 20 minute time scales. The fixed sun sensor is mounted near the top of the gondola facing the sun (rearward) direction. This sensor provides a rough measure of the telescope’s azimuth relative to the anti-sun direction, and is used during flight to determine the extent of the scanning in azimuth. The three KVH ECore2000 fiber optic rate gyroscopes allow measurement of the telescope acceleration in Azimuth, pitch, and roll. The gyroscope signals are useful for integrating across short gaps in the information from other sensors. The elevation sensor is a 16bit digital encoder, and measures the elevation of the inner gondola frame (containing the optics and receiver) with respect to the outer gondola frame.

The previously mentioned pointing sensors were all used and proven during the 1998 flight of the Boomerang telescope. For B03, two new pointed sensors were added to improve attitude reconstruction after the flight. The pointed sun sensor (PSS) is a two axis sensor that uses a four quadrant photodiode[66]. The sensor tracks the sun by using two motors to point the sensor so that the signal on all quadrants of the photodiode are balanced. The position of the sensor with respect to the telescope frame is recorded with two 16bit encoders.
The tracking star camera consists of a monochrome, Peltier cooled COHU 4920 series CCD camera attached to a Maksutov 500mm, f/5.6 telephoto lens. In this configuration, the camera has a 30’ field of view with 4 arcsecond/pixel resolution. A 715nm high pass filter was used in front of the camera, and the gain was minimized in order to avoid saturation while looking at the sky during Antarctic daylight hours. The lens was shielded by a long baffle whose interior was blackened to prevent scattered light from entering the optics. The star camera is mounted to the gondola using an equatorial mount, and is positioned using two high torque stepper motors. Two encoders are used with a PID loop (and the azimuth gyroscope) to keep the camera locked onto a star during telescope scanning. A Matrox METEOR framegrabber is used to capture video images at 10Hz. For each frame, the star’s identity and location in pixels, as well as the encoder positions, are recorded for post-flight pointing reconstruction.

3.3 The Cryogenic Cooling System

In order to achieve the desired sensitivity with our detectors, we must cool them to 0.3K for the duration our balloon flight. This requires a robust cryogenic cooling system with a long hold time. We use a closed cycle, single-shot, $^3$He fridge containing 60 liters STP of gas. A 60 liter tank of pumped $^4$He liquid at $\approx$1.7K is used to condense the $^3$He into a liquid. A charcoal adsorption pump (cryopump) is used to pump on the $^3$He liquid, cooling it down to 0.29K under nominal thermal loads.
The $^3$He fridge is designed to withstand the harsh conditions of an Antarctic balloon flight[50]. In order to achieve at least a 12 day hold time (and contain the amount of $^3$He required to achieve this), the physical size of the fridge is designed to be large enough so that the internal pressure at room temperature does not pose a safety risk. Additionally, the fridge must be able to withstand accelerations of 10g in the vertical direction and 5g at $45^\circ$ from vertical. Figure 3.2 is a sketch of the layout of the fridge. The charcoal adsorption pump is located above the condensation point, which lies above the evaporator. A thin-walled (250μm) stainless steel tube connects the pump and the evaporator to the condensation point and provides a thermal break. These tubes are offset to prevent radiative loading on the evaporator when the pump is being heated. The condensation point is thermally linked to the pumped $^4$He bath. A mechanical, motor-driven heat switch can be closed to thermally connect the charcoal pump to the $^4$He bath.

During a normal cycle, the charcoal adsorption pump starts off cold (either at 2K if a previous cycle has just ended, or at $\approx 20$K if this is the initial cool down). The heat switch is opened to thermally isolate the pump. The adsorption pump is then heated to $\approx 40$K in order to outgas any $^3$He. This gas then condenses in the center of the fridge, which is kept below 2.5K. The liquid $^3$He then drips down into the evaporator chamber. Heat is applied to the cryopump by running current through a 350 Ohm heating element that passes through the pump chamber. This heating is carried out in several pulses to allow time for the $^3$He to desorb while preventing overheating of the condensation point.
Figure 3.2: The closed-cycle BOOMERANG 300mK fridge consists of a charcoal adsorption pump and an evaporator pot which is thermally linked to the focal plane. $^3$He gas condenses at the central heat exchanger, which is thermally linked to the pumped $^4$He bath. From [63].

While applying constant power, a slowing or reversal of the cryopump temperature rise is indicative of the desorption of helium gas. After the majority of $^3$He has desorbed, the heat switch is closed, and the cryopump begins to cool. A condensation point temperature of 2K corresponds to an $\approx 80\%$ condensation rate[50]. As the charcoal
cools below ≈10K, it begins to adsorb $^3$He gas inside the fridge. This lowers the vapor pressure and causes evaporation of the $^3$He liquid. The resulting evaporative cooling lowers the temperature of the evaporator (and the attached focal plane) to below 0.3K in roughly 5 hours. A graph of a typical fridge cycle is shown in Figure 4.1.

In order to reduce the thermal load on the $^3$He system, there are several levels of thermal insulation. The fridge and entire receiver are located inside the toroidal $^4$He tank. Evaporating gas from the pumped $^4$He tank travels through a serpentine tube thermally linked to the outside of a copper cylinder surrounding the $^4$He tank. This copper shield is cooled to ≈20K by the evaporating gas. This shield is wrapped in 30 layers of aluminized Mylar (superinsulation). Outside of this lies a 65 liter toroidal tank filled with liquid nitrogen at 77K. The outside of this LN$_2$ tank is wrapped in an additional 30 layers of superinsulation. The entire system is suspended by Kevlar string inside an evacuated aluminum dewar. The window into this dewar consists of a 50μm thick sheet of polypropylene. A set of activated charcoal pellets is attached to the $^4$He tank and is used as a cryopump to maintain the vacuum space during the flight. Figure 3.3 shows a diagram of the entire cryostat. In addition to the extensive insulation, care must also be taken to prevent excess heat from entering the $^4$He stage of the cryostat along the optics path. Just inside the cryostat window, at the 77K stage, lies a 540GHz low pass metal mesh filter. A similar filter with a 450GHz cutoff is placed at the entrance to the 2K stage. Although these filters block external sources of radiation, it was determined during preflight testing that the filters themselves had a considerable
Figure 3.3: The full BOOMERANG cryostat, showing the relative positioning of the LN$_2$ and $^4$He tanks and the 20K vapor cooled shield attached to the serpentine exhaust line from the helium tank. The tanks are suspended from Kevlar rope. From [63].

emissivity when heated by external light (see section 4.4.2). After determining the source of this external thermal load, we placed an additional infrared blocking filter in-line with these 2 filters. This greatly reduced the thermal load on the $^4$He stage.

3.4 The Microwave Receiver and Optics

The optics, detectors, and associated cooling and data readout electronics are mounted to the inner frame of the gondola. This entire inner frame can be moved to change the pointing elevation of the telescope. The secondary optics and receiver are located inside the cryostat.
3.4.1 Optics

The primary mirror is a 1.3m, off-axis paraboloid with a focal length of 1280mm that is constructed out of 6061 aluminum. The light collected by this ambient temperature mirror reaches a focal point at the entrance to the cryogenic cooling system (section 3.3). The light then passes through a 2mil thick polypropylene window and expands into an aluminum box containing the secondary and tertiary mirrors, which are cooled to 2K. The surface of the secondary mirror is an ellipsoid with two equal, conjugate focal lengths of 20cm. The tertiary mirror is a paraboloid with a focal length of 33cm. It is an image of the primary, and acts as a Lyot stop to restrict the field of view of the detectors from being too close to the edge of the primary mirror. In this configuration, the primary mirror is underfilled by 50% in area. The secondary and tertiary mirrors refocus the light onto a wide focal plane containing the entrances of our feed horns (see figure 3.4). The interior surfaces of the 2K optics box are coated with a mixture of carbon lampblack and Stycast epoxy. This ensures that any beam spillover at the tertiary falls on a cold, constant, (approximate) blackbody, and that any extra optical power from this sidelobe pick-up is far less than the sky signal in the main beam [13]. A 2% transmissive Neutral Density Filter (NDF) is mounted near the focus of the primary and at the entrance of the optics box. The NDF is attached to a motor driven axle, which can be used to move the NDF into and out of the beam. With the NDF in the beam, the optical power incident on the detectors is close to the load expected at the float altitude.
Figure 3.4: A drawing of the BOOMERANG optics path. Light is gathered by the primary and focused on the window at the entrance to the cryostat. It then re-expands into the 2K “optics box” containing the secondary and tertiary mirrors and is refocused onto the entrances of the feeds. From [13].

3.4.2 Bolometers

At our frequencies of interest, cryogenic bolometers are currently the most sensitive type of detector. A general semiconductor bolometer works by exposing an absorbing material to electromagnetic radiation. The absorber has some heat capacity, $C$, and is weakly linked to a bath of some temperature $T_0$. The magnitude of this weak link ($G$) determines how rapidly the absorber can dissipate incident optical power. Under a changing optical load, the bolometer temperature changes with a time constant of approximately $\tau \approx \frac{C}{G}$. The temperature of the absorber ($T$) is measured with a calibrated thermistor. The smallest detectable signal in our bolometers is given by the “noise equivalent power” (NEP) in units of $W/\sqrt{Hz}$. Signal smaller than this level are
swamped by the noise present in the detector. The NEP is limited by Johnson noise in the thermistor, phonon noise in the weak thermal link, photon noise in the absorber, and noise in the cold JFET amplifiers (see section 3.5). The amplifier noise has been made sub-dominant by selecting low-noise JFETS. The Johnson noise is similar to the usual expression for a resistor, but with an extra term to account for the self-heating of the thermistor by the noise[52]:

\[ V_J = \sqrt{\frac{4kT}{R} (R + Z)^2 \left[1 + \omega^2 \tau^2\right]} \],

(3.1)

where \( Z \) is the dynamic impedance \((dV_{\text{bolo}}/dI_{\text{bias}})\) at the specified operating point. To convert this voltage noise into an NEP, we divide by the responsivity:

\[ NEP_J = \sqrt{\frac{4kT}{RS} (R + Z)^2 \left[1 + \omega^2 \tau^2\right]} \].

(3.2)

The phonon noise is given by [52]:

\[ NEP_{\text{phonon}} = \sqrt{4k_BT^2G}, \]

(3.3)

and the photon noise is (using the notation in [52]):

\[ NEP_{\text{photon}} = \sqrt{2h\nu Q(\nu) \left[1 + \frac{\eta e(\nu)\epsilon(\nu)}{\exp\left(\frac{h\nu}{kT}\right) - 1}\right]} , \]

(3.4)

where \( Q(\nu) \) is the spectral density of the absorbed radiation, \( \eta e(\nu) \) is the optical passband and \( \epsilon(\nu) \) is the source emissivity. The total NEP for the detector is then:

\[ NEP^2 = NEP_J^2 + NEP_{\text{phonon}}^2 + NEP_{\text{photon}}^2 . \]

(3.5)

When considering measurements of the CMB, it is more useful to consider the noise equivalent temperature (NET) in units of \( K/\sqrt{s} \). This describes the noise level in a
pixel for a given length of integration time. To get this quantity we simply take the noise voltages computed above and divide by the detector responsivity to fluctuations in the CMB. We also must convert from $\sqrt{Hz}$ to $\sqrt{s}$. The NET is

$$\text{NET} = \frac{1}{\sqrt{2}} \frac{V_{\text{noise}}}{S_{\text{CMB}}}.$$  

(3.6)

Figure 3.5 shows a simple diagram of the bolometer parts[55]. In reality, the various

![Diagram of bolometer parts](image)

Figure 3.5: This is an extremely simplified model of a bolometer. Incident radiation heats up the absorber, and the temperature change is measured by a thermistor. The magnitude of temperature increase depends on the incident power, the heat capacity of the absorber, and the thermal link to the bath.

bolometer parameters are not constant and depend on the instantaneous state of the bolometer. For example, as the temperature of the absorber increases relative to the bath, the thermal link often becomes greater. Also if the thermistor is biased with a fixed current, then the changing resistance of the thermistor changes the electrical
power that is dissipated into the absorber. Properly measuring all of the bolometer parameters involves testing the bolometer under different loading scenarios and with varying resistor bias (see section 4.4).

The B03 telescope uses a combination of Spiderweb Bolometers[5] and Polarization Sensitive Bolometers (PSB’s)[32]. Both types of detectors use an absorber consisting of a silicon nitride mesh which has been plated with approximately 120Å of gold. Neutron Transmutation Doped (NTD) thermistors are used as the temperature sensors. The mesh design of the absorbers reduces the cross-section for interactions with cosmic rays, reduces the heat capacity of the absorber, and reduces susceptibility to microphonic pickup. Each absorber is suspended from a series of unplated Si₃N₄ legs at the end of a reconcentrating feed horn. A gold plated backshort is placed at a distance of λ/4 from the absorber in order to form an integrating cavity. The PSB absorbing meshes are constructed in a grid which is metalized in only one direction, making the PSBs sensitive to only one polarization. In the case of the PSB’s, two orthogonal absorbing meshes are located in the same cavity and spaced ≈ 60μm apart in the axial direction. This colocation of two orthogonal detectors allows us to measure both polarizations of the incoming light while looking through an identical optics chain.
Figure 3.6: This figure is a photo of the type of PSB used in the 2003 flight of BOOMERANG. The diameter of the grid is 2.6mm, and the thermistor is mounted to the edge of the grid. From [32].

3.4.3 The Focal Plane

The B03 focal plane consists of eight pixels; four pairs of PSBs with acceptance bands centered at 145GHz and four photometers, each sensitive to two frequency bands centered at 245GHz and 345GHz. Each photometer pixel has a linear polarizer at its entrance to restrict its sensitivity to one polarization orientation. The entire focal plane including the re-imaging optics is thermally linked to the cold head of the $^3$He fridge by gold plated copper heat straps. The focal plane is mounted to the 2K stage of the
Figure 3.7: This figure is a photo of the type of Spiderweb Bolometer used in the 2003 flight of BOOMERANG (as well as B98). The diameter of the web is 4.0mm, and the thermistor is attached to the central pad. From [13].

cryostat via four legs made from thin walled Vespel. These legs provide the thermal break between the 2K and 0.3K stages.

Because bolometers (by definition) are sensitive to radiation over a wide range of frequencies, we must use filters to restrict our sensitivity to a specific band of interest. These filters are capacitive metal-mesh filters constructed by deposition onto polypropylene substrates[42].

Our three frequency bands were chosen to avoid emission from the diffuse atmosphere at float altitude and to provide a good measurement of the CMB and galactic foreground signals. Figure 3.8 shows an emission spectrum of the atmosphere at an
altitude of 33km, along with the designed frequency bands used by B03. Figure 3.9 shows the layout of the focal plane optics and the locations of the filters. The polarization orientation for the eight beams on the sky is shown in figure 3.10. The

![Relative Atmospheric Emission at an Altitude of 33km](image)

**Figure 3.8:** This shows the design edges of the B03 frequency bands. The actual transmission properties vary across the band and are measured prior to flight (see section 4.2).

optics within a PSB pixel consists of a corrugated “back-to-back” feedhorn which has a waveguide cutoff at 122GHz and acts as a high-pass filter. This back-to-back feed is attached to the 2K stage, and is followed by a thermal gap and then a corrugated recon-

47
centrating feed attached to the 0.3K stage. Corrugated feed horns are used because they better preserve polarization orientations- which is critical since the polarization sensitive measurements occur at the other end of the feed structure. At the entrance to the reconcentrating feed are a set of low-pass filters that define the upper edge of the band. The feed entrance is shaped to provide a uniform phase front for efficient coupling to these filters. The throat of the reconcentrating feed acts as a modal filter to block higher modes from propagating. The feed exit is impedance matched to free space[32].

Figure 3.9: The feed structures and filter arrangement for the B03 focal plane. From [49].
Figure 3.10: Diagram of the orientation of the detector beams on the sky. The W and X photometers are oriented angles of ±45° from the vertical, while the Y and Z photometers are respectively aligned vertically and horizontally. The PSBs within a pair are orthogonal, and each pair is rotated by 0°, 22.5°, 45°, and 67.5°. From [49].

The optics within each photometer pixel consists of a smooth-walled back-to-back feed horn which also acts as a high-pass waveguide filter at approximately 200GHz. A polarizer grid is attached to the entrance of the back-to-back feed. The exit of the feed is shaped to optimally couple to the 0.5” diameter lightpipe in the photometer body. At the entrance to the photometer is a 420GHz low-pass filter which defines the upper edge of the two bands. After entering the photometer, radiation is split by a 295GHz dichroic filter. Lower frequency light passes through and is measured by one spiderweb bolometer. Higher frequency light is reflected towards a second spiderweb bolometer. Both of these bolometers also have an additional low-pass filter to block any stray, out of band light.
3.4.4 Sources of Spurious Signals

Although the optical path of our detectors contains filters to limit the bandpass, it still might be possible for stray electric fields to dissipate power in the bolometer absorbers. Of particular concern are the transmitters used to communicate between the balloon and the ground station. These include the TDRSS transmitter at 1.3GHz and the ARGOS transmitter at 400MHz. To avoid this possibility, the detectors are surrounded by three Faraday cages. All electrical connections which pass through these cages have in-line filters to remove RF pickup in the wiring. The first Faraday cage is the bolometer cavity, where signal wires have 20pF capacitors to ground at the exit of the cage. The second Faraday cage consists of the 2K optics box, which is sealed with conductive aluminum tape. The electrical lines leaving this cage from the cold preamp stage pass through a series of stripline cables embedded in EV Roberts CR-124 cast Eccosorb. The third Faraday cage is the outer surface of the cryostat vacuum vessel. Signal lines leaving the cryostat pass through Spectrum 1212-0502 filters.

In addition to reducing RF pickup in the bolometers, we also take steps to reduce physical vibrations of the system which can induce spurious signals (microphonics). The web design of the bolometers reduces the mass that could potentially vibrate. All of the high impedance wires between the bolometers and the JFET amplifiers are securely tied down with nylon cord and Teflon tape. In practice, the resonant frequency of microphonic response (as measured by pounding on the outside of the cryostat) is
greater than 1kHz. This is high enough that we do not expect it to be excited by normal mechanical events during the flight.

### 3.5 Data Acquisition

The Boomerang data acquisition system is designed to control the biasing of the bolometric detectors and measure the voltage drop across the bolometers’ thermistors while reducing the introduction of electrical noise. The data acquisition system also records data from the various pointing and monitoring sensors for use in later data analysis.

An input voltage is applied to a pair of large impedance load resistors (30MΩ each for PSBs, 10MΩ for spiderweb bolometers) which provide an approximate current bias for the bolometer thermistors (See figure 3.11). The voltage before and after the thermistor is passed to a pair of matched (equal gain with magnitude near unity) JFETs (IR Labs model EJ-TIA JFET) in a source follower configuration. These JFET pairs are individually packaged in a sealed canister and are mounted to the 2K stage. Each sealed pair contains a heater resistor which is used to keep the JFETs at an operating temperature of \( \approx 100\text{K} \). The JFETs lower the output impedance before the signals traverse a series of stainless steel cables which leave the cryostat and enter a differential pre-amplifier (gain of 375) in the warm electronics box.

The JFETs and warm electronics introduce some 1/f noise in the readout chain. Because of this, the B03 bolometers are biased with a 144Hz sine wave across the load.
resistors in order to ensure that the signal bandwidth of the detector is above the 1/f knee of readout electronics. The output signal from the preamp is then filtered with a bi-quad bandpass centered at the bias frequency and with a bandwidth of 40Hz. This filtered signal is then demodulated with a lock-in amplifier using a square wave at the bias frequency. The demodulated signal then passes through a 20Hz lowpass anti-aliasing filter. This frequency is above the bolometer thermal cutoff of $\approx 10$Hz. This output is sampled at 5Hz by a 16bit Analog to Digital Converter (ADC). The signal at this point still contains a DC level that can range between 2-5 volts in amplitude. At this dynamic range, the noise about the mean level is smaller than the bitnoise of the ADC. To get around this, we apply a highpass filter at 5.6mHz and then amplify this result by a factor of 100. This AC coupled signal is then sampled at 60Hz with a 16bit ADC by the Data Acquisition System. Figure 3.11 contains a simplified schematic of this readout circuit. All signals from the detectors, pointing sensors and housekeeping sensors are stored to both a hard drive and a magnetic tape on the onboard computer. A degraded version of all data is transmitted over the TDRSS satellite network for purpose of monitoring during the flight.

3.6 Scanning Strategy

Although not part of the B03hardware per se, our in-flight scanning strategy is part of the operation of the telescope. The scanning motion allows different detectors to cross a given point on the sky at different times, which helps eliminate any errors due
Figure 3.11: Rough schematic of BOOMERANG bolometer readout circuitry. In-line RF filters (indicated by the red areas) are located at the interfaces of the 3 Faraday cages and at the exit of the external electronics box. From [13].
to transient events in one of the channels. The speed at which we scan is determined by
the physical constraints of the telescope and by the requirements that features of interest
on the sky occur at frequencies where our detectors are most sensitive (0.1Hz-1.0Hz).

Our sky scanning strategy was optimized to reduce the errors on the CMB power
spectra given the constraints imposed by our telescope’s hardware. If we make the
naive assumption of uniform sky coverage and uncorrelated noise between pixels, then
the errors on $\langle EE \rangle$ are approximated by[79]

$$
\sigma^2_{E,\ell} = \frac{2}{(2\ell + 1)f_{\text{sky}}} \left[ C_{E,\ell} + \frac{4\pi f_{\text{sky}}N_E^2}{\tau} \right]^2 ,
$$

where $N_T$ and $N_E$ are the effective noise equivalent temperatures in $T$ and $E$ of the
combined set of detectors and $\tau$ is the integration time spent uniformly covering the $f_{\text{sky}}$
fraction of the sky. For the case of the Boomerang 145GHz channels, $N_E = 2 \times N_T$
since there are eight linear detectors arranged in four orthogonal pairs, each with a
different orientation on the sky. If we use an estimate of our detector sensitivity and
flight time, and a set of power spectra based on the best models to date, then we can
calculate the uniform sky coverage needed to maximize the signal to noise ratio near the
first peak of $\langle EE \rangle$ and $\langle TE \rangle$. The signal to noise ratios for the spectra are a fairly flat
function of the sky coverage. The S/N for $\langle EE \rangle$ peaks near $f_{\text{sky}} \approx 70$ square degrees.

For $\langle TE \rangle$, the peak is near $f_{\text{sky}} \approx 1600$ square degrees.
The Boomerang telescope has a usable elevation range of 35 to 55 degrees. The telescope is designed to scan no more than 60° from the anti-sun direction. Exceeding this range could cause stray heating of the telescope. There are also constraints on the acceptable scan periods of the telescope. From the 1998 flight of Boomerang, we know that certain scan periods excite pendulations in the balloon-gondola system. The scan speed is restricted by the thermal time constants of the detectors, the mechanics of the telescope control systems, and the stability of our readout electronics.

Given the above constraints, we created a scan strategy that came as close as possible to covering a uniform “deep” region (for sensitivity to $\langle EE \rangle$) and a larger “shallow” integration region (for sensitivity to $\langle TE \rangle$). Because each change in pointing elevation perturbs the telescope, we decided to adjust the elevation no more than once per hour. With this restriction, we found that the smallest reasonable size that we could achieve for the “deep” region was $\approx 100$ square degrees. The size of our “shallow” region ($\approx 800$ square degrees) was bounded on one side by the galaxy and on the other side by the distance from the anti-sun direction.

When determining the details of the scan strategy, we simulated the scanning of the telescope based on the same “schedule file” used to actually control the telescope during the flight. These simulations produced a coverage map for a given schedule file. Since this coverage was non-uniform, we approximated the spectral errors by the sum
of the error contributions from each pixel

\[ \sigma_{E,\ell}^2 = \sum_p \frac{2}{(2\ell + 1)f_p} \left[ C_{E,\ell} + \frac{4\pi f_p n_{p,E}^2}{\tau} \right]^2 \]  

(3.9)

\[ \sigma_{X,\ell}^2 = \sum_p \frac{2}{(2\ell + 1)f_p} \left[ C_{X,\ell}^2 + \left( C_{T,\ell}^2 + \frac{4\pi f_p n_{p,T}^2}{\tau} \right) \right] \left( C_{E,\ell}^2 + \frac{4\pi f_p n_{p,E}^2}{\tau} \right) \]  

(3.10)

where \( f_p \) is the sky fraction of a typical pixel and \( n_{p,T} \) and \( n_{p,E} \) are the noise in a given pixel computed from the NET of the detectors and the integration time on the pixel.

The first four days of the flight were spent scanning over the “shallow” region, and the remainder of the flight was spent on the “deep” region. After confirming that the scanning schedule produced the desired sky coverage (and signal to noise), we adjusted slightly the scan speed so that it did not coincide with any pendulation modes discovered in B98. Twice a day, when our cmb scans were at nearly constant declination, we pointed the telescope at the galaxy and scanned over RCW38 and other galactic sources. Figure 3.12 show the predicted sky coverage based on the actual schedule file that was loaded in the flight computer. Note that it agrees well with the actual coverage shown in figure 5.3. Figure 3.13 shows the predicted spectral error bars from equations (3.9) and (3.10) using an effective NET of 70.7\( \mu K/\sqrt{Hz} \) per 7' pixel (assuming 8 x 145GHz detectors with an NET of 200\( \mu K/\sqrt{Hz} \) each) and a fiducial spectrum from the WMAP telescope.
Figure 3.12: This is the predicted sky coverage based on simulations using the “schedule file” that was created to control the telescope during flight. As the flight progresses (or as the launch gets delayed), the scan limit due to the location of the sun begins to encroach on our planned scan region. The dashed lines show the location of the 60° from anti-sun mark.
Figure 3.13: The predicted error bars on the $\langle TT \rangle$, $\langle TE \rangle$, and $\langle EE \rangle$ spectra. This plot assumes an effective NET of $70.7\mu K/\sqrt{Hz}$ per 7' pixel at 145GHz. The astute reader will note that these predictions are significantly better than the actual spectra shown in figure 5.10. Explanations are given in that section of the text.
Chapter 4

Testing Prior to Launch

In the months prior to launch, all of the telescope’s subsystems were tested individualy and as a whole. The receiver (detectors, filters, cold optics, and cold electronics) was tested inside the Boomerang cryostat in the lab. The purpose of these tests were to ensure that the receiver would provide sufficient sensitivity in each of our frequency bands in order to meet our science objectives. Once the full telescope was assembled in Antarctica, we did further testing to ensure that our far-field beams and pointing instruments were nominal.

4.1 Cryogenic Performance

The cryostat used in the B03 flight has been used in previous balloon flights. During preflight tests, the cryogenic cooling system performed as expected. The only exception was during one test where insufficient clearance between the back-to-back feedhorns
(at 2K) and the reconcentrating feeds (at 0.3K) resulted in a physical touch. This greatly increased the base temperature of the cold head and greatly reduced the hold time.

Under normal conditions, both the nitrogen and helium tanks are precooled to 77K with LN$_2$. After thermalizing, the nitrogen is purged from the helium tank by pressurizing the tank exhaust with $^4$He gas, forcing the LN$_2$ out the fill tube. The heat switch linking the $^3$He cryopump and the helium bath is opened, and the helium tank is then filled with liquid $^4$He. It takes approximately 9 hours for the helium tank and focal plane to fully thermalize to 4K. The cryopump, which is now thermally isolated, cools much slower. This makes it possible to minimize the heating of the pump on the initial cycle of the fridge.

After reaching 4K, a mechanical vacuum pump is used to evacuate the space above the liquid helium bath, which causes evaporation and cooling of the bath to 2K. This pump-down procedure is slow, in order to avoid thermo-acoustic oscillations of the helium vapor inside the circuitous exhaust tubing. Such oscillations can quickly dump power onto the helium bath and boil off a significant amount of $^4$He. Once the bath has reached 2K, the cryopump is heated to $\approx$40K. The heating of the charcoal in the cryopump causes any adsorbed $^3$He to outgas. As the released $^3$He comes into thermal contact with the bath at the central heat exchanger, it condenses and drips down into the evaporator pot. After condensing the $^3$He, the heat switch is closed and the cryopump cools to 2K. Evaporative cooling forced by the adsorption of $^3$He gas onto the charcoal
cools the evaporator to 300mK. Under normal loading conditions, the fridge has a hold
time of approximately 14 days.

Figure 4.1: A plot of the temperatures during a typical fridge cycle. During the first fridge
cycle after the initial cool-down, the cryopump has not cooled all the way to 2K before it is
heated to release any adsorbed $^{3}$He.
4.2 Frequency Bands

The three frequency bands used in B03 are defined by various filters in the receiver (see figure 3.9). Ideally, these bands would have sharp cutoffs in frequency and have a flat response across each band. In order to measure the actual frequency passbands, we used two techniques. For a range of frequencies near each band, we explicitly measured the transmission properties using a polarized Martin-Puplett Fourier Transform Spectrometer (FTS). Low frequency radiation that is far out of band is blocked by the waveguide cutoff of the back-to-back feedhorns. To check for high frequency radiation leaking into our bands, we used high pass grill filters[72]. Such a high frequency leak would be a serious problem, since the detector response to the CMB would be contaminated by higher frequency sky signal from sources such as galactic dust.

For the range of frequencies near each of our bands, we measured the transmission properties of each channel using a polarized Martin-Puplett FTS[48]. This instrument is essentially a classic interferometer, with a beam splitter, fixed mirror, and moveable mirror. The beamsplitter in this case is a wire grid polarizer with a six inch diameter and 2.5 mil wire spacing. This grid has a very flat response across our frequencies of interest. Two additional polarizers are used to polarize the input signal and select the polarization of the output.

When using this FTS with the Boomerang receiver, we placed the cryostat window over the output of the FTS (which is on the top of the FTS) and aligned its position and orientation according to the polarization direction of the desired channel and the
horizontal location that maximized the amplitude of the interferogram. A 77K LN$_2$ source (Raleigh-Jeans source at our frequencies) was placed at the input of the FTS. The movable mirror was scanned over a distance of 21.5cm, which gives us a spectral resolution of

$$R = \frac{c}{2D} = 0.698 \text{GHz}. \quad (4.1)$$

The motion of the mirror produces an interferogram. The velocity of the mirror was set to 1mm/sec, which is slow enough to ensure that the interferogram fringes occur at frequencies which our detectors can measure. The Fourier transform of the interferogram gives us the convolution of the input source (77K Raleigh-Jeans) spectrum and the detector response. Figure 4.2 shows the measured transmission spectra for the B03 channels. These are relative spectra (normalized to 1.0), and do not include the optical efficiency. Table 4.1 shows the calculated band centers for each channel.

For the grill filter test, three high-pass grill filters (with cutoffs of 250GHz, 350GHz, and 450GHz) were alternatively placed in front of the cryostat window. A chopped LN$_2$ thermal load (alternating between 77K and room temperature) was observed with each filter in place. These filters blocked in-band power for each of our detector frequency bands, and allowed a measurement of any out-of-band high frequency leaks. For the 145GHz channels, an upper limit of 0.5% was placed on the ratio of out-of-band power above 250GHz to in-band power. For the 245GHz and 345GHz channels, a similar limit was found above 450GHz. The grill filter and FTS results together give us confidence that our frequency bands are nominal.
Figure 4.2: The transmission spectra for all channels, measured with a Fourier Transform Spectrometer (FTS).

### 4.3 Response to Polarized Sources

In addition to the frequency response of our channels, we need to measure the cross-polar response of our detectors. This is simply the relative response to signals that are polarized orthogonally to the detector:

$$\epsilon = \frac{V_{\text{cross-polar}}}{V_{\text{co-polar}}}.$$  

(4.2)
Table 4.1: These frequencies are the band centers of the channels. The integrals of the spectral bandpass on either side of this frequency are equal. For this calculation, the spectrum of the Raleigh-Jeans source has been removed.

For B03 the primary source of any cross polarized response is the physical structure of the PSB mesh[32] and imperfect response of the polarizing grids in front of the photometers. The cross polar contribution can be thought of as an additional, constant, total-power response.

We measure this response by placing a chopped LN$_2$ source (alternating between 77K and room temperature) behind a slowly rotating polarizer grid. This apparatus is placed in front of the cryostat window, and the detector response is recorded as a function of polarizer angle. As we rotate the grid in front of the chopped source, the
bolometer response is modulated as

\[ V_{\text{bolo}} = V_{\text{cross-polar}} + V_{\text{co-polar}} \cos^2(\theta) \]

\[ = V_{\text{co-polar}} \left[ \epsilon + \cos^2(\theta) \right] \]

(4.3)

where \( \theta \) is the relative angle between the orientation of the grid and the detector. By measuring the minima and maxima of the modulated detector voltages we can compute the orientation direction of each channel and the cross polar response. These values for each channel are listed in table 4.2. The cross polar response is roughly one percent for the photometer channels and \( \approx 5-10\% \) for the PSBs (although 145Z1 is particularly high).

<table>
<thead>
<tr>
<th>Channel</th>
<th>Orientation (Degrees)</th>
<th>Cross Polar Response (( \epsilon ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>B145W1</td>
<td>137.1±2.0</td>
<td>0.080±0.006</td>
</tr>
<tr>
<td>B145W2</td>
<td>44.4±2.0</td>
<td>0.062±0.005</td>
</tr>
<tr>
<td>B145X1</td>
<td>178.4±2.0</td>
<td>0.055±0.013</td>
</tr>
<tr>
<td>B145X2</td>
<td>88.7±2.0</td>
<td>0.070±0.008</td>
</tr>
<tr>
<td>B145Y1</td>
<td>158.6±2.0</td>
<td>0.051±0.012</td>
</tr>
<tr>
<td>B145Y2</td>
<td>66.2±2.0</td>
<td>0.060±0.006</td>
</tr>
<tr>
<td>B145Z1</td>
<td>109.8±2.0</td>
<td>0.182±0.004</td>
</tr>
<tr>
<td>B145Z2</td>
<td>21.2±2.0</td>
<td>0.088±0.008</td>
</tr>
<tr>
<td>B245W</td>
<td>139.2±2.0</td>
<td>0.007±0.005</td>
</tr>
<tr>
<td>B245X</td>
<td>42.9±2.0</td>
<td>0.007±0.005</td>
</tr>
<tr>
<td>B245Y</td>
<td>178.7±2.0</td>
<td>0.000±0.015</td>
</tr>
<tr>
<td>B245Z</td>
<td>85.2±2.0</td>
<td>0.014±0.013</td>
</tr>
<tr>
<td>B345W</td>
<td>139.9±2.0</td>
<td>0.008±0.005</td>
</tr>
<tr>
<td>B345X</td>
<td>42.6±2.0</td>
<td>0.008±0.001</td>
</tr>
<tr>
<td>B345Y</td>
<td>178.7±2.0</td>
<td>0.004±0.014</td>
</tr>
<tr>
<td>B345Z</td>
<td>84.9±2.0</td>
<td>0.019±0.013</td>
</tr>
</tbody>
</table>

**Table 4.2:** The orientation of each detector on the sky is measured counterclockwise from the positive elevation direction. The cross polar response is the relative sensitivity to orthogonally polarized signals. These measurements were done centered along the optical axis.
4.4 Bolometer Characterization

Bolometer testing prior to flight is necessary to confirm that the detectors are sensitive enough to achieve our desired science goals. The final calibration of the detectors is performed with in-flight data. These pre-flight tests serve as a useful diagnostic for the entire receiver.

As discussed in section 3.4.2, proper characterization of our bolometers requires more detailed treatment than the previously mentioned approximations. For both spiderweb bolometers and PSB’s, we can model the voltage drop across the current-biased thermistor as a function of the Stokes parameters on the sky. This model depends on the intrinsic properties of the bolometer as well as the throughput of the optics chain and the details of the biasing:

\[
V_{\text{bolo}} = S_{\text{bolo}} \times P_{\text{opt}} \tag{4.4}
\]

where \( S_{\text{bolo}} \) is the optical responsivity of the bolometer and the other term is the optical power absorbed by the bolometer. The absorbed power is given by [32]

\[
P_{\text{opt}} = \frac{1}{2} \int d\nu \, \eta \lambda_0^2 e(\nu) \left[ (1 + \epsilon) I + (1 - \epsilon) (Q \cos(2\alpha) + U \sin(2\alpha)) \right], \tag{4.5}
\]

where \( \{I, Q, U\} \) are the Stokes parameters of the incoming light, \( \eta \) is the optical efficiency, \( \lambda_0^2 \) is the throughput of a single-moded feed, and \( e(\nu) \) is the frequency response of the band. The cross polar response is characterized by \( \epsilon \), and \( \alpha \) measures the orientation of the detector with respect to the coordinate system where the Stokes parameters are defined.
The optical responsivity depends on the fundamental bolometer properties and the temperature of the thermal reservoir. Although the spiderweb bolometers have a slight polarization sensitivity (of order a few percent), we use them only as total power detectors. The PSB’s have a small cross-polar response (5-10%) to light polarized in the orthogonal direction. In the steady state with a fixed incident optical power, the bolometer can be described by

\[ P_{\text{opt}} + P_{\text{elec}}(T) = G(T, T_0), \]  

(4.6)

where the left hand side of the equation contains terms for the optical power and the power dissipated by the bolometer thermistor. This electrical power depends on the resistance of the thermistor and hence on the bolometer temperature. On the right hand side we have indicated that the thermal conductance depends on the temperature of both the bolometer and the bath. Based on the work of Mather[52, 53], we can write the thermal conductance of a cryogenic semiconductor bolometer as

\[ G(T, T_0) = \int_{T_0}^{T} \frac{dG_0}{T_0} \frac{T^\beta}{T_0}(T_0^{\beta+1} - T_0^{\beta+1}) \frac{T_0}{T_0}\beta + 1. \]  

(4.7)

The resistance of the type of doped semiconductor thermistors used in our bolometers varies as[20, 77]

\[ R_{\text{bolo}}(T) = R_0 \exp \left[ \frac{\Delta}{T} \right]. \]  

(4.8)

From these expressions, we see that by measuring the parameters \( G_0, \beta, R_0 \) and \( \Delta \), we can completely describe the bolometer’s response to incident power. It is also clear from equations (4.6) and (4.7) that changing the optical load is degenerate with changing the
reservoir temperature. When attempting to measure in the lab any change in incident optical power, it is absolutely critical that the reservoir thermometry be stable from one run to the next.

4.4.1 Dark Loadcurves

In order to measure the various properties of a bolometer \((R_0, \Delta, G_0, \beta, R(T), \text{ etc})\), the first step is to measure a set of dark loadcurves (bolometer voltage versus DC bias current) at a variety of base (reservoir) temperatures. Some of our detectors were tested in the BOOMERANG cryostat with each detector blanked off at 300mK. Other detectors were tested in a small cryostat. The thermometers used in both cryostats were cross calibrated in the small cryostat. In both cryostats, the base temperature was varied by applying current to a heater resistor located on the 300mK stage.

In all cases, The loadcurves were measured at a series of evenly spaced bias voltages. Approximately 20 samples of the bolometer voltage were recorded for each bias step. These samples were averaged together to get a single number for each bias step. Figure 4.3 shows a characteristic family of dark loadcurves for one detector at a variety of reservoir temperatures. From this family of loadcurves, it is possible to measure all the fundamental bolometer properties. Using the bolometer resistance as a function of temperature, we can measure \(R_0\) and \(\Delta\). In the limit of low bias, the bolometer temperature approaches the reservoir temperature and the bolometer resistance is simply the slope of the I-V curve. We obtain a measure of this slope (and its variance) by doing
Figure 4.3: The top panel shows dark loadcurves measured for the 245W bolometer at a variety of reservoir temperatures. The dashed lines show roughly how the low-bias resistance of the bolometer changes with changing base temperature. The bottom panel shows the joint probability distribution for $R_0$ and $\Delta$ computed from the loadcurves in the top panel. Random values are drawn from this distribution when propagating errors through to the fits of $G_0$, $\beta$, and $P_{opt}$. 
a simple linear fit to the initial section of the loadcurve. The result of this fit is a measure-
mendment of $R_{bolo}(T_0)$ and the standard deviation, $\sigma_R$. In the low bias limit of equation 4.8, we get

$$\lim_{I_{bolo} \to 0} R_{bolo}(T_0) = R_0 \exp \left( \frac{\Delta}{T_0} \right)^{1/2}.$$  (4.9)

Using our measured values for $T_0$ and $R_{bolo}(T_0)$, we can do a nonlinear least-squares fit to the above equation and solve for $R_0$ and $\Delta$. The errors in the linear fit to the low-bias portion of the load curve are propagated into the calculation of these parameters. This is done by drawing random values of $R_{bolo}(T_0)$ from a gaussian probability distribution with width $\sigma_R$. For 15000 Monte-Carlo iterations we take this random value and compute the resulting $R_0$ and $\Delta$. These outputs are histogrammed to produce a joint probability distribution (see figure 4.3).

After obtaining these quantities, we can combine equations (4.6), (4.7), and (4.8) and rearrange the result into a form that is easier for fitting[71]:

$$P_{elec}(R_{bolo}) = X \left[ \frac{\Delta}{T_0 \left[ \log \left( \frac{R_{bolo}}{R_0} \right) \right]^2} \right]^{Y} - 1 \right] - X \left[ \frac{\Delta}{T_0 \left[ \log \left( \frac{R_{bolo}(T_0)}{R_0} \right) \right]^2} \right]^{Y} - 1 \right] - P_{opt}$$  (4.10)

where $Y = (\beta + 1)$ and $X = \frac{G_0 T_0}{Y}$. To use this equation, we take a load curve and compute $R_{bolo}$ and $P_{elec}$ at each bias point, and use the values of $R_0$ and $\Delta$ from the previous step. This load curve data is used to do a non-linear least squares fit to equation (4.10) to solve for $Y$ and $X$ (and hence $G_0$ and $\beta$).
To propagate the errors, we draw random samples from the $R_0/\Delta$ joint probability distribution and propagate these through the fitting. We then find the probability distribution of each parameter ($G_0$, $\beta$, and $P_{opt}$) by marginalizing over the others. Table 4.3 shows the measured bolometer parameters for all detectors. Obviously, for dark tests, we know that the optical power is close to zero. Even so, allowing the optical power to be a free parameter (and getting a value consistent with zero) is a strong confirmation that our technique is working.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Detector ID</th>
<th>$R_0$ (Ohms)</th>
<th>$\Delta$ (K)</th>
<th>$G_0$ (pW/K)</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B145W1</td>
<td>HFI10R</td>
<td>206.1±10.2‡</td>
<td>37.3±0.4‡</td>
<td>20.8±0.2</td>
<td>1.33±0.01</td>
</tr>
<tr>
<td>B145W2</td>
<td>HFI10L</td>
<td>206.1±10.2‡</td>
<td>37.3±0.4‡</td>
<td>15.9±0.2</td>
<td>1.28±0.01</td>
</tr>
<tr>
<td>B145X1</td>
<td>HFI09R</td>
<td>206.1±10.2‡</td>
<td>37.3±0.4‡</td>
<td>15.3±0.2</td>
<td>1.36±0.02</td>
</tr>
<tr>
<td>B145X2</td>
<td>HFI09L</td>
<td>206.1±10.2‡</td>
<td>37.3±0.4‡</td>
<td>20.6±0.2</td>
<td>1.27±0.02</td>
</tr>
<tr>
<td>B145Y1</td>
<td>HFI03L</td>
<td>206.1±10.2‡</td>
<td>37.3±0.4‡</td>
<td>15.4±0.2</td>
<td>1.40±0.02</td>
</tr>
<tr>
<td>B145Y2</td>
<td>HFI03R</td>
<td>206.1±10.2‡</td>
<td>37.3±0.4‡</td>
<td>18.4±0.2</td>
<td>1.28±0.02</td>
</tr>
<tr>
<td>B145Z1</td>
<td>HFI07L</td>
<td>206.1±10.2</td>
<td>37.3±0.4‡</td>
<td>18.1±0.2</td>
<td>1.36±0.02</td>
</tr>
<tr>
<td>B145Z2</td>
<td>HFI07R</td>
<td>206.1±10.2‡</td>
<td>37.3±0.4‡</td>
<td>21.8±0.2</td>
<td>1.17±0.02</td>
</tr>
<tr>
<td>B245W</td>
<td>B98150B1</td>
<td>153.0±22.4</td>
<td>39.2±1.1</td>
<td>52.7±0.5</td>
<td>0.816±0.035</td>
</tr>
<tr>
<td>B245X</td>
<td>B98150A</td>
<td>153.0±22.4‡</td>
<td>39.2±1.1‡</td>
<td>52.6±1.1</td>
<td>0.860±0.033</td>
</tr>
<tr>
<td>B245Y</td>
<td>B98DKB</td>
<td>153.0±22.4‡</td>
<td>39.2±1.1‡</td>
<td>50.8±1.2</td>
<td>0.913±0.038</td>
</tr>
<tr>
<td>B245Z</td>
<td>B98150B2</td>
<td>153.3±16.8</td>
<td>39.0±0.8</td>
<td>65.9±1.1</td>
<td>0.829±0.048</td>
</tr>
<tr>
<td>B345W</td>
<td>B98220A2</td>
<td>114.0±6.9</td>
<td>40.9±0.4</td>
<td>156.9±1.3</td>
<td>0.767±0.030</td>
</tr>
<tr>
<td>B345X</td>
<td>B98220B2</td>
<td>100.2±10.1</td>
<td>43.3±0.7</td>
<td>146.6±2.1</td>
<td>0.823±0.049</td>
</tr>
<tr>
<td>B345Y</td>
<td>M1-10</td>
<td>403.0±39.0</td>
<td>32.3±0.6</td>
<td>162.1±1.9</td>
<td>0.690±0.028</td>
</tr>
<tr>
<td>B345Z</td>
<td>B98220B1</td>
<td>75.4±3.0</td>
<td>47.1±0.3</td>
<td>161.4±0.5</td>
<td>0.908±0.013</td>
</tr>
</tbody>
</table>

Table 4.3: The bolometer parameters measured from dark loadcurves. The fit to the incident optical power was consistent with zero for all detectors. ‡The $R_0$ and $\Delta$ values were not measured independently for detectors B98150A and B98DKB. Instead, the values from B98150B1 were used for these detectors. In a similar fashion, the HFI07L $R_0$ and $\Delta$ values were used for all PSB’s.
4.4.2 Optical Loadcurves

After characterizing our bolometers with “dark” loadcurves, the next step is to see how they respond to incident light. Measuring the optical loading on our bolometers in a laboratory setting before flight is important to make sure there are no large extra sources of in-band loading. One such source of loading was found during these tests. The 540GHz and 450GHz lowpass filters located at the 77K and 2K stages were found to have an unusually high emissivity. The additional in-band load from the “hot” 77K filter would have been a major problem during the flight. An additional blocking filter was placed inline with the original, and this removed the extra power.

One quick and easy way to measure the incident optical power is to plot the bolometer electrical power as function of bolometer resistance. If this is done for two different optical loads, the difference between the curves (at low bias) is roughly the incident optical power. Unfortunately, for such a comparison to be accurate, the loadcurves must be measured at the same base temperature. Errors of even a few millikelvin will drastically influence the measured optical power, which is typically a few picowatts.

A more robust technique is to use equation (4.10) and the $R_0/\Delta$ values computed from dark loadcurves to solve for the incident power. If we do this with our NDF filter in place with the telescope looking at a 77K source, the loading should be close to what we expect during the flight. Table 4.4 is a list of the best fit values and errors.

Another useful quantity that can be obtained from optical loadcurves is the optical efficiency of the system (quantity $\eta$ in equation (4.5)). This is done by observing several
Table 4.4: The best fit incident optical power on the detectors with the NDF in the beam and observing a 77K load. The fridge temperature was fixed at 275mK for all tests. From this we see that the PSB’s all have less than one picowatt of loading and the photometers have several picowatts. The increased loading at 345GHz is likely due in part to multiple waveguide modes propagating in the feeds.

optical loads of known temperature and computing the difference in observed incident power. We do this with the NDF out of the beam (to remove errors due imprecise knowledge of the transmission properties of the filter), and look at the difference in loading between observing a 77K LN$_2$ source and a 90K liquid oxygen source. For this situation, we take the blackbody emission spectrum and integrate over the band measured in section 4.2 (renormalizing so that the integral of the spectrum is unity). We assume a single-moded feed in this calculation, which means that we probably underestimate the $A\Omega$ of the 345GHz channels. The effect of any higher waveguide modes then gets folded into the optical efficiency. The power difference between the

<table>
<thead>
<tr>
<th>Channel</th>
<th>Detector ID</th>
<th>Incident Power (pW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B145W1</td>
<td>HFI10R</td>
<td>0.38±0.03</td>
</tr>
<tr>
<td>B145W2</td>
<td>HFI10L</td>
<td>0.74±0.02</td>
</tr>
<tr>
<td>B145X1</td>
<td>HFI09R</td>
<td>0.81±0.02</td>
</tr>
<tr>
<td>B145X2</td>
<td>HFI09L</td>
<td>0.68±0.07</td>
</tr>
<tr>
<td>B145Y1</td>
<td>HFI03L</td>
<td>0.76±0.02</td>
</tr>
<tr>
<td>B145Y2</td>
<td>HFI03R</td>
<td>0.85±0.02</td>
</tr>
<tr>
<td>B145Z1</td>
<td>HFI07L</td>
<td>0.54±0.03</td>
</tr>
<tr>
<td>B145Z2</td>
<td>HFI07R</td>
<td>0.37±0.04</td>
</tr>
<tr>
<td>B245W</td>
<td>B98150B1</td>
<td>1.62±0.05</td>
</tr>
<tr>
<td>B245X</td>
<td>B98150A</td>
<td>1.30±0.05</td>
</tr>
<tr>
<td>B245Y</td>
<td>B98DKB</td>
<td>1.56±0.06</td>
</tr>
<tr>
<td>B245Z</td>
<td>B98150B2</td>
<td>1.29±0.04</td>
</tr>
<tr>
<td>B345W</td>
<td>B98220A2</td>
<td>2.63±0.06</td>
</tr>
<tr>
<td>B345X</td>
<td>B98220B2</td>
<td>3.41±0.07</td>
</tr>
<tr>
<td>B345Y</td>
<td>M1-10</td>
<td>6.34±0.09</td>
</tr>
<tr>
<td>B345Z</td>
<td>B98220B1</td>
<td>4.13±0.04</td>
</tr>
</tbody>
</table>
two loads is

\[ P_{\text{opt}}(90K) - P_{\text{opt}}(77K) = \frac{(1 + \epsilon)}{2} \eta \lambda_0^2 \int \frac{d\nu}{\nu} e(\nu) [B(90K, \nu) - B(77K, \nu)] , \]  

(4.11)

where \( B \) is the planck blackbody curve. The numbers used for the band center frequency and the cross-polar response are taken from tables 4.1 and 4.2. Because of the large absolute powers involved (since the NDF is out of the beam), explicit fitting of the loadcurve parameters is difficult. Instead, we use low-bias differences between the \( P_{\text{elec}} \) versus \( R_{\text{bolo}} \) curves to obtain the power difference on the left side of equation (4.11). The fridge temperatures for both curves used in a difference were identical to within 0.4mK, which gives us confidence that these differences were due only to optical loading. The calculated optical efficiency for each channel is listed in table 4.5.

### 4.4.3 Optical Transfer Function

In the previous sections, we have characterized the bolometers using fixed optical loads. Now we quantify the bolometer response to modulated optical signals. Bolo-metric detectors rely on temperature changes of an absorbing material that is weakly linked to a reservoir. It requires some finite time for this absorbing material to adjust to a changing optical load. This optical response time depends both on the heat capacity of the absorbing material and also on the throughput of the thermal link. This response time limits how fast we can scan our telescope in order to detect spatial structures of a particular size on the sky. We also need to be able to remove (deconvolve) the effects of this optical transfer function in the post-processing of our data.
Table 4.5: Table of the measured and the theoretical optical power difference between two observed loads (NDF out of beam, 77K and 90K). The ratio of the two quantities gives us the optical efficiency. Note that this calculation assumes that the feeds are single-moded. At 345GHz, the feeds likely propagate more modes, which results in the high efficiency in the table above.

In order to measure the response speed of the detectors, we simulate the optical loading at float altitude by moving the neutral density filter into the optical path. We then observe a chopped LN$_2$ source that is driven at a particular frequency. At lower chopping frequencies, the optical response of micromesh bolometers such as those used in BOOMERANG are well modelled as a single pole RC circuit[71]. Following the treatment in [30], Let us define the temperature coefficient of resistance as

\[
\alpha = \frac{1}{R_{\text{bolo}}} \frac{d R_{\text{bolo}}}{dT_{\text{bolo}}} = -\frac{1}{2} \left( \frac{\Delta}{T^3} \right)^{\frac{1}{2}},
\]

\[ \text{(4.12)} \]
the dynamic thermal conductance by

$$G_d = \frac{dG}{dT} = G_0 \left( \frac{T}{T_0} \right)^\beta,$$  \hspace{1cm} (4.13)

and the dynamic impedance as

$$Z = \frac{dV}{dI} = R \left[ \frac{G_d + \alpha P_{elec}}{G_d - \alpha P_{elec}} \right].$$  \hspace{1cm} (4.14)

Due to electrothermal feedback in our bolometers[52], the “effective” dynamic thermal conductance is

$$G_e = G_d - \alpha P_{elec} \left[ \frac{R_L}{R_L + R} \right],$$  \hspace{1cm} (4.15)

where $R_L$ is the load resistance used to create the current bias. The (DC) responsivity is then[71]

$$S(0) = \frac{dV}{dP_{opt}} = \frac{\alpha V}{G_e} \left[ \frac{R_L}{R + R_L} \right].$$  \hspace{1cm} (4.16)

The responsivity of the bolometer to incoming radiation with angular frequency $\omega$ is

$$S(\omega) = S(0) \left[ 1 + \omega^2 \tau_e^2 \right]^{-\frac{1}{2}},$$  \hspace{1cm} (4.17)

where $\tau_e$ is the effective timeconstant and depends on the heat capacity of the absorber and the effective thermal link:

$$\tau_e = \frac{C}{G_e}.$$  \hspace{1cm} (4.18)

We measured these timeconstants by observing a chopped LN$_2$ source under flight loading conditions (NDF in the beam). For a range of chopping frequencies we measured the relative response of each bolometer and fit these points to a single-pole model. This fit was very good at the low optical frequencies we are concerned with. Table 4.6 contains the best fit timeconstants for each channel.
<table>
<thead>
<tr>
<th>Channel</th>
<th>Timeconstant (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B145W1</td>
<td>0.070</td>
</tr>
<tr>
<td>B145W2</td>
<td>0.041</td>
</tr>
<tr>
<td>B145X1</td>
<td>0.050</td>
</tr>
<tr>
<td>B145X2</td>
<td>0.039</td>
</tr>
<tr>
<td>B145Y1</td>
<td>0.081</td>
</tr>
<tr>
<td>B145Y2</td>
<td>0.077</td>
</tr>
<tr>
<td>B145Z1</td>
<td>0.062</td>
</tr>
<tr>
<td>B145Z2</td>
<td>0.126</td>
</tr>
<tr>
<td>B245W</td>
<td>0.021</td>
</tr>
<tr>
<td>B245X</td>
<td>0.019</td>
</tr>
<tr>
<td>B245Y</td>
<td>0.018</td>
</tr>
<tr>
<td>B245Z</td>
<td>0.025</td>
</tr>
<tr>
<td>B345W</td>
<td>0.015</td>
</tr>
<tr>
<td>B345X</td>
<td>0.015</td>
</tr>
<tr>
<td>B345Y</td>
<td>0.012</td>
</tr>
<tr>
<td>B345Z</td>
<td>0.021</td>
</tr>
</tbody>
</table>

Table 4.6: Table of the optical bolometer timeconstants obtained from fitting chopped optical responses to a single-pole model.

4.4.4 Estimating Total Sensitivity

Once we have ensured that our optical loading is nominal, the next step is to select the amplitude of the bias current that will be used for the detectors during the flight. We do this by measuring the noise spectra and the responsivities of the detectors as a function of bias voltage. We then choose the bias point that maximizes the signal to noise ratio. At this operating point, we can then estimate the sensitivity (NET) of the detectors to the CMB signal.

The primary sources of noise in the readout chain are Johnson noise in the bolometer thermistor, phonon noise in the bolometer absorber, and noise in the cold JFETs and warm electronics. The noise contribution from the warm electronics is primarily 1/f noise.
noise due to drifts induced by thermal instability of some components. Some components were replaced before the flight to improve this thermal stability. For convenience, we express all of these noise quantities as Noise Equivalent Powers (NEPs), which are simply the voltage noise spectrum \((V/\sqrt{Hz})\) divided by the responsivity:

\[
NEP = \frac{PSD_{\text{noise}}}{S}.
\]  

(4.19)

The Johnson noise in the thermistor is given by\([52, 71]\]

\[
NEP_J = \sqrt{\frac{kT}{R}} \left( \frac{R + Z}{S} \right) \left[ 1 + \omega^2 \tau^2 \right]^{\frac{1}{4}},
\]

(4.20)

where \(\tau = C/G_d\). The phonon noise term is

\[
NEP_P = T \sqrt{4kG_d}.
\]

(4.21)

The NEP’s from all of the individual noise sources add in quadrature. In practice, we measure the total NEP of the system by observing a fixed optical load (NDF in beam, 77K source) that is similar to the background in flight. Rather than look at an external 77K load (which might change slightly over time), we blank off the cryostat window with a reflective aluminum plate, which gives us an effective 77K load through internal reflections from the LN\(_2\) stage. For this fixed load, we set the bias voltage, allow the bolometers to settle, and then record voltage timestreams for all channels. Then the bias level is changed and the process is repeated. The noise PSDs from these measurements are computed.

For the purposes of setting the flight bias, we used the calibration lamp to measure the voltage response to a flash from the lamp at each bias point. We then divide this
response by the noise level at 1Hz (which is close to the minimum) to get the signal to noise ratio for that bias level. The bias levels were set to 5mV increments from 5-55mV and then also to 75mV and 120mV (95mV) for the 145GHz (245 and 345GHz) channels. Based on these S/N measurements, the bias levels were set to 30mV for the 145GHz channels, 20mV for the 245GHz channels, and 25mV for the 345GHz channels.

At these bias levels, we now want to estimate the NET of each detector. We have already measured the noise PSDs, and now we just need to quantify the responsivity under flight loading. This is done by measuring the change in bolometer voltage when doing a single chop between a LN$_2$ load and a liquid oxygen (90K) load with the NDF in the beam. We can then compute the responsivity:

$$\frac{dV}{dP} = \frac{\Delta V}{\xi_{\text{NDF}} \eta \lambda_0^2 \int \! d\nu \, e(\nu) \left[ B(90K, \nu) - B(77K, \nu) \right]}.$$  \hspace{1cm} (4.22)

The numerator is the measured voltage difference, the denominator is the incident power difference, and $\xi_{\text{NDF}}$ is the throughput of the NDF. The transmission of the NDF has been measured in the lab to be $\approx 1.5\%$. So this gives us

$$\frac{dV}{dT_{\text{CMB}}} = \frac{dV}{dP} \frac{dP}{dT_{\text{CMB}}} = S \left[ \int \! d\nu \, \frac{dB(T, \nu)}{dT} \right]_{T=2.73},$$  \hspace{1cm} (4.23)

and we can now write down the expression for the Noise Equivalent Temperature (NET):

$$\text{NET} = \frac{1}{\sqrt{2}} \frac{PSD_{\text{noise}}}{dV/dT_{\text{CMB}}}.$$  \hspace{1cm} (4.24)
where the prefactor is due to the conversion from Hz in the PSD to seconds of integration time. Table 4.7 is a list of the NET’s for all channels. The noise level is the value at 1Hz measured looking at 77K with the NDF in the beam.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Noise (nV/√Hz)</th>
<th>$\frac{dV}{dT_{CMB}}$ (μV/K$_{CMB}$)</th>
<th>NET (μK$_{CMB}$√s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B145W1</td>
<td>22.0</td>
<td>77.8</td>
<td>200.0</td>
</tr>
<tr>
<td>B145W2</td>
<td>18.2</td>
<td>68.4</td>
<td>188.1</td>
</tr>
<tr>
<td>B145X1</td>
<td>21.4</td>
<td>68.4</td>
<td>221.2</td>
</tr>
<tr>
<td>B145X2</td>
<td>20.5</td>
<td>65.3</td>
<td>222.0</td>
</tr>
<tr>
<td>B145Y1</td>
<td>19.7</td>
<td>75.8</td>
<td>183.8</td>
</tr>
<tr>
<td>B145Y2</td>
<td>19.0</td>
<td>78.7</td>
<td>170.7</td>
</tr>
<tr>
<td>B145Z1</td>
<td>18.6</td>
<td>59.4</td>
<td>221.4</td>
</tr>
<tr>
<td>B145Z2</td>
<td>20.6</td>
<td>38.4</td>
<td>379.3</td>
</tr>
<tr>
<td>B245W</td>
<td>42.2</td>
<td>13.7</td>
<td>2178.1</td>
</tr>
<tr>
<td>B245X</td>
<td>24.6</td>
<td>41.2</td>
<td>422.2</td>
</tr>
<tr>
<td>B245Y</td>
<td>24.6</td>
<td>39.8</td>
<td>437.1</td>
</tr>
<tr>
<td>B245Z</td>
<td>25.8</td>
<td>38.6</td>
<td>472.6</td>
</tr>
<tr>
<td>B345W</td>
<td>22.3</td>
<td>33.0</td>
<td>477.8</td>
</tr>
<tr>
<td>B345X</td>
<td>23.2</td>
<td>38.1</td>
<td>430.6</td>
</tr>
<tr>
<td>B345Y</td>
<td>19.4</td>
<td>23.3</td>
<td>588.7</td>
</tr>
<tr>
<td>B345Z</td>
<td>52.5</td>
<td>36.1</td>
<td>1028.3</td>
</tr>
</tbody>
</table>

Table 4.7: Table of the measured noise level, responsivity, and calculated NET for each channel based on preflight testing. The noise level is quoted at 1Hz and was measured with a 77K load and looking through the NDF.

### 4.5 Beam Maps

Characterization of the B03 beams consisted of two sets of tests. The first was a rough measurement of the near-field beams to determine if there were any gross problems with the beam symmetry and polarization response of the cold optics. The second
set of tests were done in Antarctica, with the fully assembled telescope observing a source in the far-field.

4.5.1 Near Field Sanity Check

When doing preliminary receiver tests in the lab during the early stages of B03 development, we needed to be able to identify any significant problems in our detector beams. Without using the full telescope and primary mirror, our best option was to make maps of each detector beam in the near-field of the receiver. We mounted a 2-axis gimble to the BOOMERANG cryostat and attached to this a chopped thermal source consisting of the heating element from a standard automobile cigarette lighter. In front of this chopped source, we placed a polarizer whose orientation could be changed with a stepper motor. With this configuration, the source on the gimble could move along a constant radius of approximate one meter from the bottom of the cryostat.

We operated this gimble-mounted source in two configurations. The first was to remove the polarizing grid and sweep the source back and forth through the near field of the telescope approximately one meter outside of the cryostat window. Some maps for one channel at each frequency are shown in figure 4.4. The “hole” near the center of the beam maps is actually due to the hole in the center of the tertiary mirror where the calibration lamp is located. From these maps we got a crude, qualitative measurement that our beams are roughly symmetric. The next test was to move the gimble to a fixed location in the near field and slowly vary the polarizer orientation while chopping the
source. The gimble was then moved to a new location and the process repeated. This test allowed us to roughly determine the polarization response across the near field beam. Figure 4.7 shows the results from this test, using 9 locations in a 3x3 grid with the grid points separated by 10 degrees. From these plots we can see that the peak of the response occurs at a uniform direction across the beam, and that the cross polar response is not terrible. It should be emphasized that these tests were only designed to give a “sanity check” concerning the B03 beams during the initial testing of the receiver and before integration with the full telescope.

4.5.2 Far Field Measurements

To determine the beam widths of our final telescope configuration, we hung several sources made of eccosorb foam from a helium filled dirigible at a distance of $\approx 1.5$km from the telescope. Using an artificial source like this is necessary in Antarctica, since
Figure 4.5: This plot shows the response of a 145GHz channel to 9 fixed-location measurements in the near field of the telescope. The measurement points are located on a 3 x 3 grid with points separated by 10 degrees, and with the central grid point along the optics path. At each grid point, the chopped source was modulated by rotating a polarizer. The red lines are the best fit to equation 4.3. From these plots we can see that the polarization direction of a given channel is fairly uniform across the beam. We can also see that the cross polar response is ≈10%.

BOOMERANG cannot go to low enough elevation to scan over any planets. The telescope was scanned over these sources while keeping track of the pointing using the B03 pointing sensors. The star camera was used to track the eccentric target, and the coarse sun
Figure 4.6: This plot shows the response of a 245GHz channel to 9 fixed-location measurements in the near field of the telescope. The measurement points are located on a 3 x 3 grid with points separated by 10 degrees, and with the central grid point along the optics path. At each grid point, the chopped source was modulated by rotating a polarizer. The red lines are the best fit to equation 4.3. From these plots we can see that the polarization direction of a given channel is fairly uniform across the beam. We can also see that the cross polar response is \( \approx 10\% \).

sensor was used with a collimated spotlight (“artificial sun”). The eccosorb source was an 18” diameter ball, which corresponds to an angular size of 1.0’ at 1.5km. A gaussian
Figure 4.7: This plot shows the response of a 345GHz channel to 9 fixed-location measurements in the near field of the telescope. The measurement points are located on a 3 x 3 grid with points separated by 10 degrees, and with the central grid point along the optics path. At each grid point, the chopped source was modulated by rotating a polarizer. The red lines are the best fit to equation 4.3. From these plots we can see that the polarization direction of a given channel is fairly uniform across the beam. We can also see that the cross polar response is $\approx 10\%$.

was fit to the source crossings and beam widths measured using this technique can be found in table 4.8.
<table>
<thead>
<tr>
<th>Channel</th>
<th>Beam Width (FWHM, arcminutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B145W1</td>
<td>9.8</td>
</tr>
<tr>
<td>B145W2</td>
<td>9.8</td>
</tr>
<tr>
<td>B145X1</td>
<td>9.6</td>
</tr>
<tr>
<td>B145X2</td>
<td>9.7</td>
</tr>
<tr>
<td>B145Y1</td>
<td>10.0</td>
</tr>
<tr>
<td>B145Y2</td>
<td>10.0</td>
</tr>
<tr>
<td>B145Z1</td>
<td>9.9</td>
</tr>
<tr>
<td>B145Z2</td>
<td>9.6</td>
</tr>
<tr>
<td>B245W</td>
<td>6.2</td>
</tr>
<tr>
<td>B245X</td>
<td>6.4</td>
</tr>
<tr>
<td>B245Y</td>
<td>6.2</td>
</tr>
<tr>
<td>B245Z</td>
<td>6.2</td>
</tr>
<tr>
<td>B345W</td>
<td>7.0</td>
</tr>
<tr>
<td>B345X</td>
<td>6.7</td>
</tr>
<tr>
<td>B345Y</td>
<td>8.0</td>
</tr>
<tr>
<td>B345Z</td>
<td>6.9</td>
</tr>
</tbody>
</table>

**Table 4.8:** Table of the measured beam widths calculated from gaussian fits to source crossing of an eccosorb target in the far field.
Chapter 5

Data Analysis

On January 6th, 2003, the B03 telescope was launched from McMurdo Station in Antarctica. The flight lasted 311 hours and was terminated on January 21st. The altitude of the telescope was not as stable as desired, due to a small leak in the balloon. Ballast drops on day three and day five of the flight allowed some recovery of altitude. The track of the telescope did not follow the typical circumpolar pattern. This was due to instability in the upper atmosphere wind patterns as well as the lower than normal altitude. Despite the less than ideal conditions, the attitude scanning and elevation control performed as expected until the balloon fell below an altitude of 25 kilometers near the end of flight. At this point the telescope was shut down in preparation for termination. Figures 5.2 and 5.1 show plots of the altitude and track of the telescope. A total of 119 hours were spent scanning over the “deep” observation region, 79 hours were spent on the “shallow” region, and the galactic plane was observed for 30 hours.
A plot of the actual sky coverage from the flight is shown in figure 5.3, and can be

Figure 5.1: A plot of the track of the B03 telescope over the Antarctic continent (original by B. Crill). Alternating days after launch are plotted in red and black. The instability of the circumpolar winds and the lower than expected altitude led to an incomplete circumnavigation of the continent. The flight was terminated in a remote region of Antarctica and only the data vessel was recovered that year. A recovery expedition the following year returned any other equipment of value.

compared to the predicted coverage in figure 3.12. During the flight, a problem was noticed with the two tracking pointing sensors (star camera and pointed sun sensor). When both of these sensors were powered on, a data buffer on the flight computer became overloaded, causing the computer to crash. To work around this problem, the pointed sun sensor was turned off initially and the star camera was used for the first third of the flight. At this point the telescope was at a lower (and colder) altitude and
Figure 5.2: A plot of the telescopes altitude over the course of the flight. The diurnal variations are due to the variable solar heating of the gas in the balloon. The large upward jump corresponds to the main ballast drop.

the star camera froze up. At this point the pointed sun sensor was enabled for the second third of the flight. After the star camera warmed up and began functioning again, the pointed sun sensor was disabled for the remainder of the flight. The cryogenic cooling system performed nominally. The $^3$He in the refrigerator boiled off on day 11, and was cycled in order to obtain 19 more hours of data before the end of the flight.

The analysis of data from a complex experiment such as BOOMERANG is an involved process. The data timestreams from each detector must be cleaned and have the electronic and optical transfer functions of the system removed. Each timestream must then be combined with the various pointing sensors on the telescope in order to give us
Figure 5.3: This map shows the relative per-pixel integration time for all 145GHz detectors combined. Note that this does not include relative noise weighting between channels.

a measure of the sky signal plus instrument noise at each time interval. Given this Time Ordered Data (TOD) stream for each channel, we must estimate the noise component in each one. Using the TODs and an estimate of the noise, we can construct maps of the sky and the noise covariance between pixels.

Estimating cosmological parameters from CMB data requires exploring a many-dimensional parameter space and measuring the likelihood of the parameter values given the measured data (see section 5.10). Computing these likelihoods directly from the sky maps is not feasible on current computing hardware. Instead, we make the reasonable assumption that the sky signal arose from Gaussian random processes. This assumption allows us to measure the angular power spectra of the sky maps and use these spectra in the likelihood calculations without loss of cosmologically relevant information [7][8].
This compression of the data volume is critical for analyzing modern CMB datasets such as those from Boomerang.

5.1 Flight Data Calibration

When determining the sensitivity of our detectors in flight, we use several methods. Over the course of the flight, the sensitivity of a given detector may drift. This change is recorded as changes in the detector’s response to firings of the calibration lamp, and is removed in post processing. The relative calibration between all of the channels is important because we are combining timestreams from these detectors when constructing the map. This relative calibration is computed by taking an uncalibrated temperature map for each channel and computing the angular cross power spectrum of each permutation. The ratio of this power spectrum is compared to the cross spectrum with channel 145W1 to obtain the calibration relative to that channel. More details can be found in [64]. To obtain an absolute calibration for the 145GHz channels, we make a maximum likelihood map from all of the (relatively calibrated) channels, and two other maps consisting of the W+X channels and the Y+Z channels. We then measure the angular cross spectra of the B03 data with data from the WMAP experiment. The absolute calibration is then given by the ratio of cross spectra:

\[
S = \frac{\langle a_{\ell,m}^{WX} F_{\ell}^{WX} a_{\ell,m}^{YZ} F_{\ell}^{YZ} \rangle}{\langle a_{\ell,m}^{WXZ} F_{\ell}^{WXZ} a_{\ell,m}^{WMAP} F_{\ell}^{WMAP} \rangle},
\] (5.1)
where $F$ is the transfer function which includes the effects of the mapmaking process and the effects of different angular resolutions between BOOMERANG and WMAP [49]. The cross spectrum in the numerator is between two halves of the focalplane rather than just the auto spectrum of the full focalplane. This reduces any bias due to correlated noise between detectors. Using these techniques, we obtain the absolute calibrations for the 145GHz detectors in table 5.1.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Calibration Factor (V/K_{CMB})</th>
</tr>
</thead>
<tbody>
<tr>
<td>B145W1</td>
<td>11.840 ± 0.200</td>
</tr>
<tr>
<td>B145W2</td>
<td>11.248 ± 0.190</td>
</tr>
<tr>
<td>B145X1</td>
<td>11.603 ± 0.197</td>
</tr>
<tr>
<td>B145X2</td>
<td>8.655 ± 0.147</td>
</tr>
<tr>
<td>B145Y1</td>
<td>11.035 ± 0.187</td>
</tr>
<tr>
<td>B145Y2</td>
<td>10.727 ± 0.182</td>
</tr>
<tr>
<td>B145Z1</td>
<td>7.874 ± 0.133</td>
</tr>
<tr>
<td>B145Z2</td>
<td>4.996 ± 0.085</td>
</tr>
</tbody>
</table>

**Table 5.1:** Table of the absolute calibration factors for each detector, as determined from angular cross spectra between channels and with the WMAP data.

## 5.2 Pointing Reconstruction

During the flight, the fixed sun sensor (FSS), differential GPS, and gyroscopes performed nominally. However, a problem was discovered that affected the star camera and the pointed sun sensor (PSS). When both of these sensors were online, the flight computer was unable to keep up with the data volume from the sensors. This resulted in a buffer overflow, which required a reboot of the system. To work around this problem, we used only one of these sensors at a time. The star camera was used for the first and
last third of the flight. At lower altitudes (just before the ballast drop), the star camera froze up and the PSS was used for the middle third of the flight.

After the flight, the timestreams from the pointing sensors were despiked and small gaps were filled with a linear interpolation. Large gaps were flagged for exclusion from further processing. The FSS was calibrated using average azimuth values from the star camera and GPS. The star camera and PSS were recalibrated by minimizing correlations between the orthogonal azimuth and elevation directions. At low frequencies (<50mHz) in the final pointing solution, the star and sun positions from the star camera, FSS, and GPS velocity were used to determine the best fit azimuth, pitch, and roll. At higher frequencies and across gaps, the integrated gyroscope acceleration is used. The estimated final pointing errors are $\approx 2.5'$ rms in azimuth and $\approx 1.5'$ rms in elevation[49].

### 5.3 Data Cleaning and Pre-Processing

The raw data acquired by the Boomerang telescope has been effectively convolved with a transfer function consisting of the thermal response of the detector and the filtering of the readout electronics. It also contains spikes caused by cosmic rays and our calibration lamp, and thermal events caused by elevation changes. Before using this data for CMB science, we must remove these effects.

At each step of the data cleaning, we flagged (marked as bad) some samples in the datastream and then filled these flagged samples with a constrained realization of the
noise. This noise filling consisted of using linear prediction to replace flagged samples based on the unflagged data on either side. After removing very large cosmic rays, we flagged a minimal number of samples around each elevation change and fit out a decaying exponential to the signal recovery. Next, the data streams were deconvolved with the combined thermal and electrical transfer functions determined by pre-flight lab measurements. This deconvolved (but not de-spiked) timestream was an intermediate data product that was saved for future use.

Next we removed “spike-like” features from the TODs. The samples containing signals from our calibration lamp were flagged. Medium amplitude cosmic rays were then flagged by passing a simple template through the timestream and looking for spikes larger than a certain size. The data was then heavily band-pass filtered to make small cosmic rays more visible. A second template was used to flag these small cosmic rays. One concern was that this process might result in false positives by mistaking bright galactic sources for small cosmic rays. To avoid this, we made a list of all bright source crossings in our data streams by using the coincidence of sources in both detectors of a pixel pair. These samples were explicitly unflagged. After building up a list of flagged samples, we returned to the deconvolved TODs and filled these regions with constrained realizations of the noise. Figure 5.4 shows an example of the final cleaning on a small piece of one channel. After completing this primary data cleaning, it was found that each of our timestreams contained an extraneous component that was well correlated with the accelerations of the telescope’s pitch and roll gyroscopes. We
Figure 5.4: This shows the effects of cleaning on a small segment of the 145W1 channel. The high frequency noise in the cleaned data (red) has be blown up due to the bolometer deconvolution- but this occurs at frequencies outside our signal bandwidth. The black areas represent samples that have been flagged and are not used in the final analysis. Note the telescope elevation change which creates the large feature on the left. Only the samples close to this event are flagged, and the recovery period is fit to a decaying exponential and subtracted.

fit these accelerations to the data in hour long chunks and subtracted this contribution from the timestreams.

5.4 Noise Covariance and Map Making

In order to describe how we construct a sky map from our TODs, we begin with some definitions of quantities in the time and pixel domains. At each time increment,
our data consists of some sky signal and some noise:

\[ d_t = s_t + n_t. \]  

(5.2)

The sky map made from our data is a sum of the true sky signal and a noise map due to the accumulated instrument noise in each pixel:

\[ D_p = S_p + N_p. \]  

(5.3)

For the purposes of this discussion, we treat \( p \) as a generalized pixel index which runs over \( I, Q, \) and \( U \) “pixels”. This formalism is useful if, for example, we wanted to use different pixelizations for the intensity and polarization. The conversion between the time and pixel domain is carried out by a “pointing matrix” which describes how the \( I, Q, \) and \( U \) pixels should be combined to form the value at a given time sample. In an experiment like B03 which has linearly polarized detectors, the signal at a given time depends simply on the sky pixel where the detector is looking, on the orientation of the polarizer (bolometer mesh), and on the level of cross-polar response. For example, an \textit{extremely} small dataset of 4 time samples and 3 pixels (and ignoring cross polar response) would have a pointing matrix like this:

\[
A_{pt} = \begin{pmatrix}
0 & 1 & 0 & 0 & \cos(2\alpha_0) & 0 & 0 & \sin(2\alpha_0) & 0 \\
1 & 0 & 0 & \cos(2\alpha_1) & 0 & 0 & \sin(2\alpha_1) & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & \cos(2\alpha_2) & 0 & 0 & \sin(2\alpha_2) \\
0 & 1 & 0 & 0 & \cos(2\alpha_3) & 0 & 0 & \sin(2\alpha_3) & 0
\end{pmatrix}.
\]  

(5.4)
This is essentially the same matrix discussed in section 5.6. We can now express the measured timestream data in terms of the underlying sky signal:

\[ d_t = A_p t S_p + n_t. \]  \hspace{1cm} (5.5)

We assume that the noise arises from Gaussian random processes, and we calculate the time domain noise covariance according to the method in appendix A using the estimated noise TODs described in section 5.5:

\[ N_{tt'} = \langle n_t n_{t'}^T \rangle. \]  \hspace{1cm} (5.6)

Although each detector’s noise covariance is an \( n_{\text{samples}} \times n_{\text{samples}} \) matrix (with typically sixty million samples for B03), this matrix is extremely sparse and (neglecting inter-channel noise correlations) has only a narrow, symmetric band about the diagonal. The band width is determined by timestream filtering which removes long time-lag correlations. Further, for each stationary chunk of approximately one hour, the corresponding sub-block of the matrix is Toeplitz, and may be embedded in a circulant matrix. So in practice, we only need to keep track of the first \( n_{\text{correlation}} \) elements of the first row of the matrix. We can then construct the inverse \( (N_{tt'}^{-1}) \) easily in the Fourier domain[70].

Given these representations for the timestream data and the noise covariance, we can write an expression for the maximum likelihood estimate of the sky signal. In terms of the time domain noise covariance, the probability distribution of the timestream noise is

\[ P(n_t) \propto \exp \left( -\frac{1}{2} \left[ n_t^T N_{tt'}^{-1} n_t + \text{Tr}(\ln N_{tt'}) \right] \right). \]  \hspace{1cm} (5.7)
Substituting equation (5.5) into this, we obtain the probability of obtaining our measured data for a given signal map:

\[ P(d_t | S_p) \propto \exp \left( -\frac{1}{2} \left[ (d_t - A_{pt}S_p)^T N_{tt'}^{-1} (d_t - A_{pt}S_p) + \text{Tr}(\ln N_{tt'}) \right] \right). \] (5.8)

If we maximize the log likelihood of this with respect to the signal map, we obtain an expression for the most likely data map given our measured TOD

\[ D_p = (A_{pt}^T N_{tt'}^{-1} A_{pt})^{-1} A_{pt}^T N_{tt'}^{-1} d_t. \] (5.9)

The inverted portion is the pixel domain noise covariance:

\[ N_{pp'} = (A_{pt}^T N_{tt'}^{-1} A_{pt})^{-1}. \] (5.10)

The other term is known as the “noise weighted map”:

\[ Z_p = A_{pt}^T N_{tt'}^{-1} d_t. \] (5.11)

Solving equation (5.9) by brute force involves building and inverting a \( n_{\text{pixels}} \times n_{\text{pixels}} \) matrix. This inversion is carried out by Cholesky factorization using the SCALAPACK distributed linear algebra package. This calculation is done by the MADPING program, which is one of the MADCAP analysis tools[9]. In some cases, the inverse pixel domain noise covariance is poorly conditioned, and this inversion must be done in other ways. See Appendix B for further discussion of these issues. If one does not require the full pixel domain noise covariance, then it is possible to solve (5.9) iteratively. This iterative approach is used by the MADMAP program[3] (also part of MADCAP).
Another issue we must consider before making maps of our TOD’s is that for a
given pixel resolution, some pixel values in our signal map are not well defined. This
can be due to insufficient coverage of the pixel (poor signal to noise), and/or not enough
crossings at a variety of polarizer angles. The latter case results in poor separation
of the polarized and non-polarized signal. If these pixels with unconstrained signal
components are included in the analysis, it can further increase any numerical issues.

For our analysis, we first create a mask of pixels which we consider “non-degenerate”.
We set an arbitrary threshold of 100 total hits per pixel for the combined datasets. We
then use the pointing data for each timestream and sum up the 9 elements of the sym-
metric pointing matrix for each sky pixel:

\[
\mathcal{P}_{\text{sky}} = \frac{1}{n} \sum \begin{pmatrix}
(1 + \epsilon)^2 & (1 - \epsilon^2) \cos(2\alpha) & (1 - \epsilon^2) \sin(2\alpha) \\
\cdots & (1 - \epsilon)^2 \cos^2(2\alpha) & (1 - \epsilon)^2 \cos(2\alpha) \sin(2\alpha) \\
\cdots & \cdots & (1 - \epsilon^2) \sin^2(2\alpha)
\end{pmatrix}
\]

(5.12)

where the sum is over \( n \) hits in the sky pixel. We can see that if this 3x3 matrix has any
singular values, then we cannot solve for the individual \( I, Q, \) and \( U \) components. We
call a pixel with such a matrix “degenerate”. For practical purposes, even matrices that
are “close” to singular have signal components that are not constrained well enough.

As a measure of how well conditioned the matrix is, we compute the determinant. The
maximum possible value of the determinant is 0.25, and we use 0.21 as an empirically
determined threshold. All pixels with a determinant larger than this are considered
non-degenerate. After creating this mask of good pixels, we go through each signal
timestream and flag any samples lying outside this region.

5.5 Noise Estimation

We have seen that in order to construct the maximum likelihood map given our data,
we need to have an estimate of the time domain noise covariance. The basic method
employed to estimate the noise is an iterative approach. We begin with the assumption
that the TODs are noise dominated. Our first estimate of the noise TOD is simply the
data itself. For each iteration, we do the following:

1. Given an estimate of the noise TODs, fill all flagged samples with a constrained
   realization of the noise.

2. Given these noise TODs, construct the inverse time-domain noise covariance for
each piece-wise stationary chunk of the data. Because this matrix is Toeplitz
and can be embedded in a circulant matrix, we only need to compute the first
row of the matrix. The inverse noise covariance can be multiplied by a time-
domain vector by convolution in the frequency domain. For this reason, the noise
covariance is often called a “noise filter”.

3. Fill flagged samples in the data TOD’s with the corresponding values in the esti-
   mated noise TOD.
4. Given our flag-filled data TOD’s and a set of piece-wise noise filters, use MADmap to make I, Q, and U maps. These maps are an estimate of the sky signal.

5. Using the pointing matrix, go through each data TOD and subtract out the estimated sky signal for the pixel which is being observed at that sample. The result is an improved estimated of the noise TOD’s.

6. Repeat this process until further iterations do not affect the resulting noise TOD’s and noise filters.

In practice, after two or three iterations the noise TOD’s have converged to better than a percent.

5.6 Motivation for Using PSB Sums and Differences

The B03 focal plane contains four pairs of orthogonal PSBs. Each detector in a pair sees identical incident power from the telescope optics. In order to exploit this arrangement for common mode rejection, we use sums and differences of the timestreams from a pair of detectors. The timestream voltage of a single PSB is (assuming the integral over the passband is one):

\[ V = \frac{S_{\text{CMB}}}{2} \eta \lambda_0^2 \left[ (1 + \epsilon)I + (1 - \epsilon)(Q \cos(2\alpha) + U \sin(2\alpha)) \right] \] (5.13)

where \( S_{\text{CMB}} \) is the voltage responsivity to CMB temperature fluctuations in units of \( V/\mu K \). Before combining timestreams, we calibrate each one individually and convert
to $\mu$K. The sum and difference timestreams between the two bolometers is then

$$B_{\text{sum}} = \left[1 + \frac{\epsilon_1 + \epsilon_2}{2}\right] I + \frac{1}{2} \left[ (1 - \epsilon_1) \cos(2\alpha_1) + (1 - \epsilon_2) \cos(2\alpha_2) \right] Q \nonumber$$

$$+ \frac{1}{2} \left[ (1 - \epsilon_1) \sin(2\alpha_1) + (1 - \epsilon_2) \sin(2\alpha_2) \right] U \quad (5.14)$$

$$B_{\text{diff}} = \left[\frac{\epsilon_1 - \epsilon_2}{2}\right] I + \frac{1}{2} \left[ (1 - \epsilon_1) \cos(2\alpha_1) - (1 - \epsilon_2) \cos(2\alpha_2) \right] Q \nonumber$$

$$+ \frac{1}{2} \left[ (1 - \epsilon_1) \sin(2\alpha_1) - (1 - \epsilon_2) \sin(2\alpha_2) \right] U, \quad (5.15)$$

For purposes of illustration, if we make the simplifying assumption that we can neglect the cross polar response, then we have

$$B_{\text{sum}} = I + \frac{1}{2} \left[ \cos(2\alpha_1) + \cos(2\alpha_2) \right] Q \nonumber$$

$$+ \frac{1}{2} \left[ \sin(2\alpha_1) + \sin(2\alpha_2) \right] U \quad (5.16)$$

$$B_{\text{diff}} = \frac{1}{2} \left[ \cos(2\alpha_1) - \cos(2\alpha_2) \right] Q \nonumber$$

$$+ \frac{1}{2} \left[ \sin(2\alpha_1) - \sin(2\alpha_2) \right] U, \quad (5.17)$$

so one of the main benefits of this technique is that the intensity data is mostly isolated in the sum TODs. This has the effect of minimizing leakage of power in $I$ into $Q$ and $U$. For experiments with excellent crosslinking (each pixel is observed at many different polarizer orientations), this leakage is not as pronounced[33]. For BOOMERANG, we can compare the leakage when using PSB sums and differences to the leakage when using each channel individually. We do this by simulating sky maps containing only CMB signal, using input fiducial power spectra based on the best-fit WMAP cosmology and making use of the “synfast” utility from the HEALPix[17] software package. We then
take these signal-only maps and scan over them with the B03 scan strategy in order to produce signal-only TODs.

Once we have these pure signal TODs for each independent channel, we make a second dataset which contains the sums and differences of the TODs. Next we need a set of real noise filters for each dataset. We obtain these from the actual B03 data using the procedure in section 5.5, and carrying out the noise estimation procedure for both independent and sum/difference TODs. The final step in this test is to make maps of the pure signal TODs using the actual B03 noise filters. This is done using the same MADmap program as in the noise estimation procedure. The output maps are then differenced with the original input maps and the residual from both methods is compared. Figure 5.5 shows the residual leakage when using independent channels compared to sum and difference TODs. Clearly for the B03 scan strategy and noise properties, using sums and differences of the PSB timestreams results in less leakage of $I$ power into $Q$ and $U$.

## 5.7 The Angular Power Spectra

Once we have created polarized maps of the CMB and our noise, we want to be able to determine the most likely model of the universe that could have given rise to this measurement of the CMB. This process involves a Bayesian likelihood analysis (see section 5.10). This process scales as $O(N^3)$, where $N$ is the number of datapoints that must be varied (number of pixels in this case) [73]. For large maps, this process...
Figure 5.5: Maps were made from pure-signal TODs using the real B03 noise filters. The original input maps were then subtracted. The residuals are plotted here for the cases of treating each channel independently versus using sums and differences. We can see that for B03, sums and difference are much better at preventing leakage of temperature into polarization.

is essentially impossible. It is therefore necessary to compress the cosmological information contained in the map into some other form with fewer data values, and without losing information. This is accomplished by assuming that the radiation field is generated by Gaussian random processes, which implies that the statistical information in
the map is entirely contained within the angular power spectra [7]. The power spectra have far fewer data points, making a likelihood analysis possible.

Once we have maps of the sky, we can proceed to compute the six angular power spectra of these maps (see section 2.1.2). For full-sky, noise-free maps this can be done with a direct expansion in spherical harmonics. In the presence of noise and when using a small patch of the sky, we need a way to quantify our errors on the estimates of the power spectra. There are two widely used techniques for carrying out this procedure. The first uses simulations of the full B03 dataset to perform a Monte Carlo analysis of the errors[26]. In other words, the spherical harmonic transform of the cut sky map is computed directly, and the effects of the finite sky coverage, the scan strategy, and time domain processing of the data are removed by Monte Carlo analysis of ensembles of realizations of signal and noise maps. The second technique uses the full pixel-domain noise covariance to compute the spectral errors.

5.8 Monte Carlo Results

The Monte Carlo method applied to B03 data has been published previously[31, 62, 56], and here we give a brief overview of this technique and its results. The first step is to estimate the time domain noise correlations following the procedure in 5.5. The Monte Carlo analyses additionally include any inter-channel noise correlations due to cross-talk. A map of the data is made using the time domain noise correlations and an iterative approach to solving equation (5.9). Note that this does not use the
full pixel-pixel noise covariance matrix. A direct calculation of the spherical harmonic transform is carried out on this partial-sky map. The angular power spectrum computed by this method will differ from the “true” power spectrum of the underlying CMB. This difference is due to the sky cut, the smoothing effects of the detector beams, the noise in the timestream, and any filtering which has been done.

Consider first the case of a signal-only, full sky map. If we compute the power spectrum of this map and the spectrum with our sky cut applied, the two are related via a “coupling kernel.” More specifically, if we do this many times, the ensemble averages are related by [26]:

$$\langle \tilde{C}_\ell \rangle = \sum_{\ell'} K_{\ell\ell'} \langle C_{\ell'} \rangle.$$  

(5.18)

The coupling kernel can be computed analytically based on the spherical harmonic transform of the pixel domain window used to cut and weight the data. Next we consider the effects of noise, filtering and beams. These features can be described for a single spectrum by:

$$\tilde{C}_\ell = \sum_{\ell'} K_{\ell\ell'} B_{\ell'}^2 F_{\ell'} C_{\ell'} + \tilde{N}_\ell.$$  

(5.19)

In this case $B_{\ell\ell}$ is the beam window function, $N_{\ell}$ is the power spectrum of the noise, and $F_{\ell}$ is the transfer function that describes the effect of all filtering, etc. The transfer function is estimated by creating an ensemble of signal only maps which are realizations consistent with a fiducial spectrum. These maps are scanned with the B03 scanning strategy to produce signal timestreams. The real filtering is applied and each set of timestreams is made into a map and a then a cut-sky spherical harmonic trans-
form is computed. Comparing this power spectrum to the input gives us the $F_\ell$ transfer function. Next, noise-only timestreams containing the same spectral properties as the real noise are simulated. These timestreams are made into noise maps and the power spectrum is computed. The ensemble average of these noise power spectra give us $N_\ell$ above. We can now solve for our estimate of the underlying sky signal angular power spectrum.

In order to estimate the errors on the power spectra and their binwise corellations (necessary for parameter estimation), we start with our estimate of the underlying spectrum and generate a new, full set of signal plus noise Monte Carlo realizations. The binned power spectrum is computed and we compute the bin-bin covariance matrix:

$$C_{bb'} = \langle (C_b - \bar{C}_b)(C_{b'} - \bar{C}_{b'}) \rangle. \quad (5.20)$$

The angular power spectra computed by the Monte Carlo method (which was used in section 5.10) is plotted in figure 5.6.

### 5.9 MADCAP Results

The Microwave Anisotropy Dataset Computational Analysis Package (MADCAP) is a suite of parallel software designed to compute the angular power spectra of CMB data using a “maximum likelihood” method [9]. This process involves two steps, the first of which is to take the measured data stream and an estimate of the time domain noise covariance and compute the pixel domain noise covariance matrix and the maxi-
Figure 5.6: The version of the Monte Carlo based angular power spectra that was used for the cosmological parameter estimation in section 5.10. Figure from [47].

...maximum likelihood map of the sky. This is done with the MADPING program, which does a brute force solution of equations (5.10) and (5.11), and multiplies these to obtain the maximum likelihood data map. The second step uses this map and the pixel domain noise covariance to compute the maximum likelihood angular power spectra. Since the processes generating the signal and noise are independent, the pixel-pixel data covariance is equal to the sum of the signal covariance and the noise covariance:

\[
D_{pp'} = S_{pp'} + N_{pp'}.
\] (5.21)
Where the pixel index in this case runs over T, Q, and U “meta-pixels”. The correlations in these matrices are arranged in blocks like this:

\[
S_{pp'} \propto \begin{pmatrix}
\langle T_p T_{p'} \rangle & \langle T_p Q_{p'} \rangle & \langle T_p U_{p'} \rangle \\
\langle Q_p T_{p'} \rangle & \langle Q_p Q_{p'} \rangle & \langle Q_p U_{p'} \rangle \\
\langle U_p T_{p'} \rangle & \langle U_p Q_{p'} \rangle & \langle U_p U_{p'} \rangle \\
\end{pmatrix}.
\]  

(5.22)

The signal covariance above depends entirely on the geometrical distribution and shape of the pixels, on the detector beams, and on the underlying CMB power spectra being considered:

\[
S_{pp'} = R_{pp'} \sum_\ell W_{\ell,p} W_{\ell,p'} C_{\ell} M_{\ell,pp'}.
\]  

(5.23)

In the above expression, \( R_{pp'} \) is a quantity that takes into account the rotation of the Stokes parameters at each pixel onto the great circle which connects those pixels. The \( M_{\ell,pp'} \) parameter depends on the angular separation between the pixels. The \( W \) quantities are the combined beam and pixel window functions for a given pixel. Typically we divide the power spectra into bands in \( \ell \). The spectrum at any given \( \ell \) is the product of a continuous “shape function” and the value of the bin in which it lies:

\[
C_{\ell} = C_b C_{\text{shape}(\ell)}.
\]  

(5.24)

The signal covariance can hence be written as a sum over each bin of the constant bin value times the partial derivative with respect to that bin:

\[
S_{pp'} = \sum_b C_b \frac{\partial S_{pp'}}{\partial C_b}.
\]  

(5.25)
Using equation 5.23, the partial derivative of the signal covariance then becomes

$$\frac{\partial S_{pp'}}{\partial C_b} = R_{pp'} \sum_{\ell \in b} W_{\ell,p} W_{\ell,p'} C_{\text{shape},\ell} M_{\ell,pp'}.$$ \hspace{1cm} (5.26)

The exact forms of $R_{pp'}$ and $M_{\ell,pp'}$ depend on the types of pixels being correlated and to which of the six power spectra the bin belongs. Recall from section 2.1 that we can expand the $T$, $Q$, and $U$ maps in terms of spherical harmonics. Using the spin raising and lowering operators, we can rewrite equations 2.5-2.7 as

$$T(\theta, \phi) = \sum_{\ell m} a_{T,\ell m} Y_{\ell m}(\theta, \phi)$$ \hspace{1cm} (5.27)

$$Q(\theta, \phi) = \frac{1}{2} \sum_{\ell m} \left( a_{2,\ell m} \left[ \frac{(\ell - 2)!}{(\ell + 2)!} \right]^{\frac{1}{2}} \bar{\delta}^2 Y_{\ell m}(\theta, \phi) + a_{-2,\ell m} \left[ \frac{(\ell + 2)!}{(\ell - 2)!} \right]^{\frac{1}{2}} \bar{\delta}^2 Y_{\ell m}(\theta, \phi) \right)$$ \hspace{1cm} (5.28)

$$U(\theta, \phi) = \frac{i}{2} \sum_{\ell m} \left( a_{2,\ell m} \left[ \frac{(\ell - 2)!}{(\ell + 2)!} \right]^{\frac{1}{2}} \bar{\delta}^2 Y_{\ell m}(\theta, \phi) - a_{-2,\ell m} \left[ \frac{(\ell + 2)!}{(\ell - 2)!} \right]^{\frac{1}{2}} \bar{\delta}^2 Y_{\ell m}(\theta, \phi) \right).$$ \hspace{1cm} (5.29)

When we compute the correlations between the fields in equations (5.27)-(5.29), we will end up with terms involving correlations between $a_{T,\ell m}$, $a_{2,\ell m}$, and $a_{-2,\ell m}$, as well as some (very tedious) functions of $\ell$, $m$, and $\theta$. If we carry this out and substitute the definitions for the power spectra given in (2.25)-(2.30), we arrive at the following expressions for each of the six types of spectral bins in equation (5.26). In order to avoid confusion, we label the pixels with I, Q, and U ($p$ is the column index and $p'$ the
\[
\frac{\partial S_{pp'}}{\partial C_b \in TT} = R_1 \sum_{\ell \in b} \frac{2\ell + 1}{4\pi} W_{\ell,p} W_{\ell,p'} P_{\ell} (z_{pp'}) C_{\text{shape},\ell}^{TT} (5.30)
\]

\[
\begin{cases}
R_1 \rightarrow 1 & \text{for } p = I, p' = I \\
R_1 \rightarrow 0 & \text{for all other blocks}
\end{cases}
\]

\[
\frac{\partial S_{pp'}}{\partial C_b \in EE} = \sum_{\ell \in b} \frac{2\ell + 1}{4\pi} W_{\ell,p} W_{\ell,p'} \left[ R_1 F_{\ell}^{12} (z_{pp'}) + R_2 F_{\ell}^{22} (z_{pp'}) \right] C_{\text{shape},\ell}^{EE} (5.31)
\]

\[
\begin{cases}
R_1, R_2 \rightarrow \cos (2\alpha_{p'}) \cos (2\alpha_{p}), -\sin (2\alpha_{p'}) \sin (2\alpha_{p}) & \text{for } p = Q, p' = Q \\
R_1, R_2 \rightarrow -\sin (2\alpha_{p'}) \cos (2\alpha_{p}), -\cos (2\alpha_{p'}) \sin (2\alpha_{p}) & \text{for } p = Q, p' = U \\
R_1, R_2 \rightarrow -\cos (2\alpha_{p'}) \sin (2\alpha_{p}), -\sin (2\alpha_{p'}) \cos (2\alpha_{p}) & \text{for } p = U, p' = Q \\
R_1, R_2 \rightarrow \sin (2\alpha_{p'}) \sin (2\alpha_{p}), -\cos (2\alpha_{p'}) \cos (2\alpha_{p}) & \text{for } p = U, p' = U \\
R_1, R_2 \rightarrow 0, 0 & \text{for all other blocks}
\end{cases}
\]

\[
\frac{\partial S_{pp'}}{\partial C_b \in BB} = \sum_{\ell \in b} \frac{2\ell + 1}{4\pi} W_{\ell,b} W_{\ell,b'} \left[ R_1 F_{\ell}^{12} (z_{pp'}) + R_2 F_{\ell}^{22} (z_{pp'}) \right] C_{\text{shape},\ell}^{BB} (5.32)
\]
\[
\begin{align*}
R_1, R_2 \rightarrow \sin (2\alpha_{p'}) \sin (2\alpha_p), & \quad \text{for } p = Q, p' = Q \\
R_1, R_2 \rightarrow \cos (2\alpha_{p'}) \sin (2\alpha_p), & \quad \text{for } p = Q, p' = U \\
R_1, R_2 \rightarrow \sin (2\alpha_{p'}) \cos (2\alpha_p), & \quad \text{for } p = U, p' = Q \\
R_1, R_2 \rightarrow \cos (2\alpha_{p'}) \cos (2\alpha_p), & \quad \text{for } p = U, p' = U \\
R_1, R_2 \rightarrow 0, & \quad \text{for all other blocks}
\end{align*}
\]

\[
\frac{\partial S_{pp'}}{\partial C_{\ell_b} \in TE} = R_1 \sum_{\ell \in b} \frac{2\ell + 1}{4\pi} W_{\ell,p} W_{\ell,p'} F_{\ell}^{10} (z_{pp'}) C_{\text{shape},\ell}^{TE} \quad (5.33)
\]

\[
\begin{align*}
R_1 \rightarrow \cos (2\alpha_{p'}) & \quad \text{for } p = I, p' = Q \\
R_1 \rightarrow -\sin (2\alpha_{p'}) & \quad \text{for } p = I, p' = U \\
R_1 \rightarrow \cos (2\alpha_p) & \quad \text{for } p = Q, p' = I \\
R_1 \rightarrow -\sin (2\alpha_p) & \quad \text{for } p = U, p' = I \\
R_1 \rightarrow 0 & \quad \text{for all other blocks}
\end{align*}
\]

\[
\frac{\partial S_{pp'}}{\partial C_{\ell_b} \in TB} = R_1 \sum_{\ell \in b} \frac{2\ell + 1}{4\pi} W_{\ell,p} W_{\ell,p'} F_{\ell}^{10} (z_{pp'}) C_{\text{shape},\ell}^{TB} \quad (5.34)
\]
\[
\begin{align*}
R_1 & \rightarrow \sin (2\alpha_{p'}) \quad \text{for } p = I, p' = Q \\
R_1 & \rightarrow \cos (2\alpha_{p'}) \quad \text{for } p = I, p' = U \\
R_1 & \rightarrow \sin (2\alpha_p) \quad \text{for } p = Q, p' = I \\
R_1 & \rightarrow \cos (2\alpha_p) \quad \text{for } p = U, p' = I \\
R_1 & \rightarrow 0 \quad \text{for all other blocks}
\end{align*}
\]

Here \(R_{pp'}\) has been written out in terms of functions involving \(\alpha\), which is the angle to rotate the meridian passing through the specified pixel \((p\) or \(p')\) onto the great circle connecting both pixels. \(P_\ell\) is the standard (spin-zero) Legendre polynomial, and the

\[
\frac{\partial S_{pp'}}{\partial C_{b \in EB}} = R_1 \sum_{\ell \in b} \frac{2\ell + 1}{4\pi} W_{\ell,p} W_{\ell,p'} \left[ F_{\ell}^{12}(z_{pp'}) + F_{\ell}^{22}(z_{pp'}) \right] C_{shape,\ell}^{EB} \quad (5.35)
\]
functions $F_{10}^\ell$, $F_{12}^\ell$, and $F_{22}^\ell$ follow the appendix of [75] and are given by:

\[
F_{10}^\ell(z) = 2 \frac{\ell z}{1-z^2} P_{\ell-1}(z) - \left[ \frac{\ell}{1-z^2} + \frac{\ell(\ell-1)}{2} \right] P_\ell(z) \tag{5.36}
\]

\[
F_{12}^\ell(z) = 2 \frac{(\ell+2)z}{1-z^2} P_{\ell-1}^2(z) - \left[ \frac{\ell-4}{1-z^2} + \frac{\ell(\ell-1)}{2} \right] P_\ell^2(z) \tag{5.37}
\]

\[
F_{22}^\ell(z) = 4 \frac{(\ell + 2) P_{\ell-1}^2(z) - (\ell - 1) z P_\ell^2(z)}{\ell(\ell-1)(\ell+1)(\ell+2)(1-z^2)} \tag{5.38}
\]

where $P_\ell^2$ is the spin-two Legendre polynomial. For a given set of power spectra, we can compute the signal covariance (and hence the data covariance). The probability of our data map given these spectra is then

\[
P(D_p|C_\ell) \propto \exp \left( -\frac{1}{2} \left[ D_p^T D_p^{-1} D_{p'} + \text{Tr}(\ln D_{pp'}) \right] \right). \tag{5.39}
\]

The purpose of the MADSPEC program is to find the spectra which maximize this probability. We can write the log-likelihood of the power spectra (less some overall constant) as

\[
\mathcal{L}(C_\ell) = -\frac{1}{2} \left( D_p^T D_p^{-1} D_{p'} + \text{Tr}(\ln D_{pp'}) \right). \tag{5.40}
\]

To maximize this log likelihood, we assume that near the peak of the likelihood we can approximate its shape by a quadratic function. We then use the Newton-Raphson method to iteratively solve for the zero of the derivative of the likelihood. At each iteration, a correction is applied according to the approximation that the likelihood is quadratic:

\[
\delta C_{i+1} = \left( \left[ \frac{\partial^2 \mathcal{L}}{\partial C^2} \right]^{-1} \frac{\partial \mathcal{L}}{\partial C} \right)_{C=C_i} \tag{5.41}
\]
So given our data map, an estimate of the pixel-space noise covariance, and an initial guess at the power spectra, we can iteratively solve for the spectral values which maximize the likelihood in equation (5.40).

If we look at equations (5.26), (5.40), and (5.41) above, we can see we need to calculate the bin-wise derivatives of the signal covariance once, and then multiply them by the current spectral values and sum them during each iteration of the spectral calculation. In practice, we often just recalculate these rather than use up a significant amount of disk space. For some combinations of pixel locations, the signal covariance matrix as computed above will be poorly conditioned. If the noise is very small, the resulting data covariance may be impossible to invert by Cholesky factorization. See Appendix B for more information.

Initial analysis of the full 145GHz B03 data with MADCAP showed a large noise bias in the jackknife (null test discussed below) of the $\langle EE \rangle$ spectrum. This was likely due to the large inter-channel noise correlations (from wiring cross-talk) which are not accounted for in MADCAP. For this analysis, we conservatively chose to use only the half of the detectors which have the smallest inter-channel correlations (the W and X PSB pairs).

After general data cleaning and flagging of degenerate pixels, the sum and difference timestreams were computed. The TODs were bandpass filtered from 0.07Hz to 14.5Hz with a brick-wall filter and then reformatted into the native binary input format used by MADCAP. In this format, pointing information consists of the HEALPix pixel
index and a weight for I, Q, and U. These pointing weights account for the orientation and cross-polar response of the detectors—essentially these weights are the coefficients in equations (5.14) and (5.15). We then chose contiguous “chunks” of the data over which the noise was likely to be piecewise-stationary. These chunks were chosen to lie in between changes in telescope elevation, which resulted in 187 chunks of roughly one hour each for each TOD. Collections of chunks separated by large time gaps (such as those when scanning the galaxy) were grouped into data “intervals”. Using the noise estimation techniques discussed previously, the noise TOD was estimated for the combination of all channels using maps with a HEALPix NSIDE resolution of 512. The final inverse time-time noise correlations for each chunk were saved for use in estimating the pixel-pixel noise covariance. All flagged timesamples in the detector TODs were replaced by the value in the corresponding estimated noise TOD. Figure 5.7 shows a plot of the final signal plus data map and the residual map computed from the noise TODs using the PCG mapmaker (MADMAP). For the CMB power spectrum analysis all pointing files were initially pixelized into HEALPix pixels with an NSIDE resolution of 512. Because the Boomerang data set consists of a “deep” sky region with higher signal-to-noise and a “shallow” region with lower S/N, it is beneficial to use two different pixelization resolutions for the two regions. This greatly reduces the needed computing resources while still making good use of the available S/N. This 2-resolution approach was implemented by creating a pixel space mask of the deep region at the desired resolution of the shallow region. Our deep region mask contains 1729 pixels at
Figure 5.7: The right hand plots show maps of the estimated noise TODs after 4 iterations of subtracting an estimate of the signal TOD from the data TODs. The left hand plots show the PCG maps created from the data TODs using the final time-time noise correlations produced by the estimation process. Note that the data maps have had an additional pixel mask applied, which is why the coverage is slightly different.

A 14 arcminute resolution. For each TOD, we have three “pointing files”, one for each of I, Q, and U. Each pointing file contains the HEALPix pixel number and the weight for the specified component. Using our pixel mask, we split each pointing file into two separate files - one for the deep region and one for the shallow. The pixel value of each timesample in the original pointing file was compared to the deep region mask.
and placed in one of the two new pointing files. The pixel value in the other pointing file was flagged. Pixels added to the shallow pointing file were degraded to the desired resolution.

This multi-resolution data set containing four signal TODs and six pointing files each was used as input, along with the chunkwise inverse time-time noise covariances, into the MADPING program. The main outputs of this code are the pixel domain accumulated inverse noise covariance and the noise weighted map. These data products consist of 20523 total I/Q/U metapixels at 7 arcminute resolution and 64914 metapixels at 14 arcminute resolution.

For B03, the polarization information in the shallow scan region is insignificant due to the extremely low S/N. To save computational resources, we cut all shallow Q and U metapixels from the analysis. We remove these pixels from the noise weighted map and cut the corresponding rows and columns of the inverse pixel-pixel noise covariance. This is equivalent to marginalizing over the Q and U shallow pixels. Next we apply a mask to the shallow intensity pixels to trim the edges of the map. This reduces our final pixel count to 20523 deep metapixels and 12980 shallow metapixels (33503 total metapixels).

Before using this matrix for spectrum estimation we computed the eigenspectrum. As can be seen in figure 5.8, this matrix is poorly conditioned. As we will see below, in order to estimate the power spectra, we need to compute the inverse square root of this matrix and also be able to add the signal covariance and invert the result. In
Figure 5.8: The eigenvalues of the pixel domain noise covariance. Note the small eigenmodes which result in the matrix being poorly conditioned.

In principle, this can all be done using Cholesky factorization. In our experience, the poor conditioning of the matrices and the closely spaced eigenvalues can sometimes lead to the failure of this factorization. As a more robust solution, we compute the full eigendecomposition of all matrices using the {SCALAPACK} “divide and conquer” function, {PDSYEV}. We then take the vector of eigenvalues, modify as necessary to create the inverse, inverse square root, etc, and recompose the new matrix from the eigenvectors. From our noise covariance, we can take the noise weighted map and compute the maximum likelihood data map:

\[ D_p = N_{pp'} Z_{p'} . \]  \hspace{1cm} (5.42)
This final map is shown in figure 5.9. Note the higher resolution of the central deep scan region. Although MADSPEC is not yet formally designed to handle multiple resolutions within a single dataset, it is possible to process such a dataset with a minor workaround. Under normal usage, two datasets will have no pixel-pixel correlations between them. Given multiple datasets, MADSPEC constructs a global map (a concatenation of the individual maps) and a global pixel noise covariance (a block diagonal matrix with

**Figure 5.9:** The maximum likelihood maps constructed from the noise-weighted map and the pixel-pixel noise covariance. These are the maps which were used in the estimation of the power spectra.
the blocks consisting of the individual noise covariances). It then block distributes
this global matrix amongst all of the processors. The main feature we exploit is that
MADSPEC will not block distribute the pixel noise covariance if the block distributed
files already exist. So we take our multi-resolution noise covariance (which contains
non-zero correlations between pixels of different resolutions) and “pre-distribute” it
before running MADSPEC. We then treat the maps at differing resolutions as separate
datasets, each with their own pixel and beam window functions.

We have also extended MADSPEC in several ways. The first is to carry out all
calculations in the S/N eigenbasis using simultaneous diagonalization by means of a
Karhunen-Loeve transform[6]. In this basis, our data covariance has eigenmodes whose
magnitude corresponds to the S/N in that mode. The way we actually do this is to
precompute the regularized $N^{-1/2}_{pp'}$ matrix. Then we use this matrix to transform each
derivative signal covariance matrix:

$$\frac{\partial S_{pp'}}{\partial C_b} = N^{-1/2}_{pp'} \frac{\partial S_{pp'}}{\partial C_b} N^{-1/2}_{pp'}.$$

(5.43)

In this basis, the noise covariance is simply the identity matrix, and the likelihood
expression in equation (5.40) becomes

$$D_p^* = N^{-1/2}_{pp'} D_p$$

(5.44)

$$D^*_{pp'} = S^*_{pp'} + I$$

(5.45)

$$L(C_\ell) = -\frac{1}{2} \left(D_p^* T D^*_{pp'}^{-1} D_p^* + \text{Tr}(\ln D^*_{pp'}) \right).$$

(5.46)
The second modification is to use an eigendecomposition and regularization when inverting the data covariance. Since we are in the S/N eigenbasis, this is equivalent to cutting modes which have poor S/N. In the case of the B03 dataset, this regularization is only needed when far from the convergence values of the spectra.

The spectral binning for this analysis was chosen to match the binning of the earlier Monte Carlo analyses. The ell-space window function used for each pixel of a given resolution was obtained by convolving the beam and pixel window functions. The final spectrum estimation required 10 iterations and took approximately 10 hours on 2500 AMD Opteron cores running at 2.6GHz. The spectra are plotted in figure 5.10, and show the results of earlier Monte Carlo analyses, as well as the best fit spectra for the WMAP ΛCDM model. From this plot of the spectra, we see several interesting features. Although we have used less data than the Monte Carlo method (which used all eight 145GHz channels), our error bars are smaller across many ranges of multipole ℓ. This is a result of using the full pixel-pixel noise covariance in the spectrum estimation. Unfortunately, it is also clear that at mid to high ℓ there is a residual bias in the ⟨EE⟩ and ⟨BB⟩ spectra. This bias is likely due to the fact that when constructing the noise covariance with the MADPING program, we have ignored inter-channel noise correlations. Although this greatly reduces the computational cost of building the inverse pixel domain noise covariance, it is clearly not sufficient for a fully unbiased calculation of ⟨EE⟩ and ⟨BB⟩ at high ℓ. This incomplete description of the noise and the fact
Figure 5.10: Plot of the six angular power spectra as computed by the MADSPEC program. Also plotted are the WMAP ΛCDM best fit spectra and the previously published [31, 62, 56] Monte Carlo analyses of the same raw data (using the FASTER analysis tools).

that only half the channels were used likely accounts for the discrepancy between the measured spectral error bars and the predictions in figure 3.13.
5.9.1 Systematics Tests and Possible Explanations

In order to test the validity of our power spectra results, there are several checks we can do. One of the most robust is to compute a “jackknife” test by computing the power spectra from the difference of two halves of our data. These power spectra should be consistent with zero. To accomplish this, we first divide the timestreams into pieces. The first half of the shallow region scans and the first half of the deep region scans form the first piece and the remainder forms the second piece. We then make maps and noise covariance matrices from both halves of the data. The difference map and difference noise covariance is then given by

\[ \Delta_p = \frac{D_{\text{first}}^p - D_{\text{second}}^p}{2} \]  
\[ \Delta N_{pp'} = \frac{N_{\text{first}}^{pp'} - N_{\text{second}}^{pp'}}{4} \]

The spectra computed from this first-half minus second-half jackknife are plotted in figure 5.11. This jackknife test confirms the noise bias in polarization for values larger than about \( \ell = 800 \). In the \( \langle TT \rangle \) jackknife, it may seem that some of the spectral bins have failed the test. These failures, however, are very small compared to the value of the spectrum in those bins. Based on this test we are confident in our results, with the exception of the high \( \ell \) polarized spectra.
Figure 5.11: Plot of the six angular power spectra jackknives described in this section and computed by the MADSPEC program. The failures in $\langle TT \rangle$ are very small compared to the value of the actual spectrum in those bins. The high $\ell$ $\langle BB \rangle$, $\langle TE \rangle$, and $\langle EB \rangle$ spectra exhibit a noise bias.

5.10 Impact on Cosmological Parameters

As we have seen earlier in this chapter, the maximum likelihood power spectra computed using the MADCAP tools are largely consistent with the previously published
Monte Carlo based analyses. For Gaussian random initial perturbations, the angular power spectra contain all relevant cosmological information. Using a set of canonical B03 power spectra, we can estimate the likelihood of a set cosmological parameters given our data. The probability density function (PDF) of the posterior of the cosmological parameters given our data is:

$$P(\theta | C_\ell) \propto P(\theta_{\text{prior}}) P(C_\ell | \theta),$$

(5.49)

where $\theta$ are the cosmological parameters and we have included the probability of the priors imposed on the parameters. For a given set of cosmological parameters, the CAMB [45] software is used to evolve the primordial state specified by those parameters into a set of angular power spectra as observed today. The probability of our observed spectra relative to this set of (binned) model spectra is given by the chi-squared of the two spectra with the bin-bin correlation matrix (Fisher information matrix):

$$\chi^2 = \sum_{bb'} [C_b(\theta) - C_b(\text{data})] F_{bb'} [C_{bb'}(\theta) - C_{bb'}(\text{data})].$$

(5.50)

For B03, a Monte-Carlo Markov Chain (MCMC) method was used (in the CosmoMC [44] software package) to explore the cosmological parameter space and determine the maximum likelihood parameters for the B03 dataset in conjunction with data from WMAP and other astrophysical measurements. Full details are reported in [47]. Here we summarize briefly those results.

The current “standard model” of cosmology consists of a universe which is spatially flat and whose composition is dominated by dark energy and cold dark matter.
This standard model has been built up from other CMB observations, nucleosynthesis calculations, and studies of supernovae and large scale structure (among other astrophysical measurements). Within the context of this model, we can determine how well the B03 results independently confirm this paradigm. The six cosmological parameters considered and their flat-prior values are (following the notation used in [47]):

\[ 0.5 \leq n_s \leq 1.5 : \text{Scalar spectral index} \]
\[ 2.7 \leq \ln(10^{10}A_s) \leq 4.0 : \text{Scalar amplitude} \]
\[ 0.005 \leq \Omega_b h^2 \leq 0.1 : \text{Proportional to baryon density} \]
\[ 0.01 \leq \Omega_c h^2 \leq 0.99 : \text{Proportional to dark matter density} \]
\[ 0.5 \leq \theta \leq 10.0 : \text{Represents the position of the peaks in the angular power spectra} \]
\[ 0.01 \leq \tau \leq 0.8 : \text{Thomson scattering optical depth from present to decoupling} \]

Additionally, there are some implied weak priors on other values. The age of the universe is assumed to be between 10 and 20 billion years, and \( H_0 \) is between 45 and 90.

As an initial consistency check, the B03 temperature spectrum and polarization spectra were separately used to find the maximum likelihood values of the parameters in the baseline set above. Those results are consistent, which gives us confidence in the polarization data. Next, the full B03 data was combined with the WMAP results[25, 36] and then with a set of other CMB experiments and large scale structure data. The results of these combinations can be found in table 5.2. From this we see that the B03+WMAP
alone case is nearly as powerful as the case of adding several additional CMB experiments. In addition to the baseline set of parameters, another set of tests was carried out.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>B03</th>
<th>WMAP</th>
<th>B03+WMAP</th>
<th>B03+ALL</th>
<th>B03+ALL+LSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_b h^2$</td>
<td>$0.0218^{+0.0029}_{-0.0030}$</td>
<td>$0.0245^{+0.0021}_{-0.0019}$</td>
<td>$0.0241^{+0.0017}_{-0.0016}$</td>
<td>$0.0233^{+0.0013}_{-0.0012}$</td>
<td>$0.0227^{+0.0008}_{-0.0008}$</td>
</tr>
<tr>
<td>$\Omega_c h^2$</td>
<td>$0.128^{+0.020}_{-0.026}$</td>
<td>$0.111^{+0.010}_{-0.016}$</td>
<td>$0.109^{+0.013}_{-0.010}$</td>
<td>$0.106^{+0.010}_{-0.010}$</td>
<td>$0.120^{+0.006}_{-0.005}$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$1.055^{+0.010}_{-0.010}$</td>
<td>$1.048^{+0.007}_{-0.007}$</td>
<td>$1.049^{+0.005}_{-0.005}$</td>
<td>$1.045^{+0.004}_{-0.004}$</td>
<td>$1.045^{+0.004}_{-0.004}$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$&lt; 0.49$</td>
<td>$0.23^{+0.14}_{-0.11}$</td>
<td>$0.22^{+0.11}_{-0.10}$</td>
<td>$0.170^{+0.009}_{-0.008}$</td>
<td>$0.106^{+0.047}_{-0.048}$</td>
</tr>
<tr>
<td>$n_s$</td>
<td>$0.86^{+0.10}_{-0.10}$</td>
<td>$1.02^{+0.06}_{-0.06}$</td>
<td>$1.01^{+0.06}_{-0.05}$</td>
<td>$0.98^{+0.04}_{-0.03}$</td>
<td>$0.95^{+0.02}_{-0.02}$</td>
</tr>
<tr>
<td>$\log [10^{10} A_s]$</td>
<td>$3.4^{+0.5}_{-0.2}$</td>
<td>$3.4^{+0.2}_{-0.2}$</td>
<td>$3.3^{+0.2}_{-0.2}$</td>
<td>$3.2^{+0.2}_{-0.2}$</td>
<td>$3.1^{+0.1}_{-0.1}$</td>
</tr>
<tr>
<td>$\Omega_A$</td>
<td>$0.65^{+0.14}_{-0.19}$</td>
<td>$0.76^{+0.07}_{-0.07}$</td>
<td>$0.77^{+0.06}_{-0.06}$</td>
<td>$0.77^{+0.05}_{-0.05}$</td>
<td>$0.70^{+0.03}_{-0.03}$</td>
</tr>
<tr>
<td>Age (GYr)</td>
<td>$13.4^{+0.6}_{-0.5}$</td>
<td>$13.3^{+0.3}_{-0.4}$</td>
<td>$13.3^{+0.3}_{-0.3}$</td>
<td>$13.5^{+0.2}_{-0.3}$</td>
<td>$13.6^{+0.2}_{-0.2}$</td>
</tr>
<tr>
<td>$\Omega_m$</td>
<td>$0.35^{+0.19}_{-0.17}$</td>
<td>$0.24^{+0.07}_{-0.07}$</td>
<td>$0.23^{+0.06}_{-0.06}$</td>
<td>$0.23^{+0.05}_{-0.05}$</td>
<td>$0.30^{+0.05}_{-0.03}$</td>
</tr>
<tr>
<td>$\sigma_8$</td>
<td>$0.97^{+0.18}_{-0.17}$</td>
<td>$0.94^{+0.12}_{-0.12}$</td>
<td>$0.92^{+0.09}_{-0.09}$</td>
<td>$0.83^{+0.06}_{-0.06}$</td>
<td>$0.85^{+0.05}_{-0.05}$</td>
</tr>
<tr>
<td>$z_{\text{recomb}}$</td>
<td>$22.0^{+11.3}_{-11.9}$</td>
<td>$19.6^{+6.6}_{-6.7}$</td>
<td>$19.3^{+5.6}_{-5.7}$</td>
<td>$16.5^{+5.1}_{-5.1}$</td>
<td>$12.6^{+3.9}_{-4.0}$</td>
</tr>
<tr>
<td>$H_0$</td>
<td>$70.4^{+19.6}_{-25.4}$</td>
<td>$76.3^{+13.7}_{-4.2}$</td>
<td>$76.7^{+13.5}_{-3.9}$</td>
<td>$75.8^{+5.0}_{-5.1}$</td>
<td>$69.6^{+2.4}_{-2.4}$</td>
</tr>
</tbody>
</table>

**Table 5.2:** Table of maximum likelihood cosmological parameters from [47] for the baseline model and including various combinations of datasets. The “ALL” dataset includes data from WMAP $\langle TT\rangle$[25] and $\langle TE\rangle$[36], DASI $\langle TT\rangle$[22], VSA $\langle TT\rangle$[15], ACBAR $\langle TT\rangle$[40], MAXIMA $\langle TT\rangle$[23] and CBI $\langle TT\rangle$[65]. The Large Scale Structure (LSS) data include galaxy power spectra from the 2dFGRS[61] and SDSS[74] surveys.

where one other parameter was allowed to vary as well. If the curvature ($\Omega_k$) is allowed to vary from the flat case with a uniform prior of $-0.3 \rightarrow 0.3$, then the resulting value using B03 and the other CMB data is $-0.037^{+0.033}_{-0.039}$. So this is very consistent with a flat universe and justifies our original baseline choice of spatially flat models. If the parameter set is expanded to allow basic CDM isocurvature modes in addition to the dominant adiabatic ones, we find that the contribution of these isocurvature modes is at most a few percent.
5.11 Where Do We Stand?

The 2003 flight of Boomerang successfully measured the polarization of the Cosmic Microwave Background radiation. The temperature and polarized angular power spectra computed from this data using a maximum likelihood approach are quite consistent with those computed using Monte-Carlo based techniques. The improvement in error bars on the angular power spectra demonstrates the usefulness of maximum likelihood methods for polarized CMB data analysis in the case of relatively low pixel counts. The slight noise bias in the high $\ell$ polarization spectra clearly illustrates the importance of accurately accounting for all detector noise properties when constructing the pixel domain noise covariance matrix. The cosmological parameters estimated from the B03 power spectra are consistent with other CMB experiments and astrophysical measurements.

Since the initial publishing of the B03 results, several other polarization results have been released. Of special note are the DASI three year results[43], QUad[80], and the three-year WMAP results[24, 58, 69]. All of these experiments present a consistent picture in which the inflationary period just after the big bang was adiabatic. In the near future, the Planck satellite promises to provide the best high resolution, full-sky measurement of CMB polarization for many years to come.

As is often the case in science, precision cosmological measurements using the CMB leave us with large questions. The nature of the observed dark matter and dark
energy is an active area of current research and, as usual, great experimental challenges offer the potential for great rewards.
Bibliography


Appendix A

Time Domain Noise Covariance

In order to compute the pixel-space noise covariance, we first need a chunk-wise measure of the time domain noise covariance (actually its inverse). While the math behind this process is relatively simple, the “devil is in the details”. This is especially true when the noise dominates the signal, such as in the $B03 \langle EE \rangle$ spectrum. The goal of this appendix to go through the details of this calculation and its implementation using the popular fast Fourier transform software FFTW (REF). There are a variety of conventions used when defining such things as the power spectrum. I have chosen my convention based on the following criteria:

- For white noise, the integral of the power spectrum should equal the variance
- For white noise, the zero-lag inverse noise correlation should equal $1/\text{variance}$

These two conditions require us to choose a particular set of definitions.
A.1 The Discrete FFT

Let us begin with defining the FFT. For the forward transform we have

\[
Y(f_k) = \sum_{j=0}^{N-1} y(t_j) \exp\left[-\frac{2\pi i j k}{N}\right] \quad k = 0, \ldots, N - 1
\]  

(A.1)

and for the inverse transform we have

\[
y(t_j) = \frac{1}{N} \sum_{k=0}^{N-1} Y(f_k) \exp\left[\frac{2\pi i j k}{N}\right] \quad j = 0, \ldots, N - 1
\]  

(A.2)

Note that the frequencies for \(k = N/2 + 1\ldots N - 1\) are negative frequencies. Also of interest is the discrete form of Parseval’s theorem

\[
\sum_{j=0}^{N-1} |y(t_j)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |Y(f_k)|^2
\]  

(A.3)

A.2 The Power Spectrum

It’s generally agreed that the power spectrum is proportional to the square of the FFT of the data. So really the only question is how is it normalized. For our purposes, we will take the power spectrum to be normalized to the “mean squared amplitude” of the original function

\[
\sum_{k=0}^{N/2+1} P(f_k) = \frac{1}{N} \sum_{j=0}^{N-1} |y(t_j)|^2
\]  

(A.4)

where \(P\) is the power spectrum, which is only defined for positive frequencies. So now the question is: what coefficients in the terms of the power spectrum are necessary to
make this hold? The answer is the following:

\[ P(0) = \frac{1}{N^2} |Y(0)|^2 \]  \hspace{1cm} (A.5)

\[ P(f_k) = \frac{1}{N^2} \left[ |Y(f_k)|^2 + |Y(f_{N-k})|^2 \right] \quad (k = 1, \ldots, \left(\frac{N}{2} - 1\right)) \]  \hspace{1cm} (A.6)

\[ P(f_{N/2}) = \frac{1}{N^2} |Y(f_{N/2})|^2 \]  \hspace{1cm} (A.7)

The coefficient of \(1/N^2\) can be seen from multiplying both sides of Parseval’s equality by \(1/N\) and then applying this to the normalization criterion. In our case, we are typically dealing only with real functions in the time domain, so the above expression can be written as

\[ P(0) = \frac{1}{N^2} |Y(0)|^2 \]  \hspace{1cm} (A.8)

\[ P(f_k) = \frac{2}{N^2} |Y(f_k)|^2 \quad k = 1, \ldots, \left(\frac{N}{2} - 1\right) \]  \hspace{1cm} (A.9)

\[ P(f_{N/2}) = \frac{1}{N^2} |Y(f_{N/2})|^2 \]  \hspace{1cm} (A.10)

### A.3 The Power Spectral Density

The Power Spectral Density (PSD) is the amount of “power spectrum” per unit frequency. If we integrate the PSD over some bandwidth, we get the average power spectrum value in that band. Often we express the PSD in units of \(Volts/\sqrt{Hz}\), instead of \(Volts^2/Hz\):

\[ PSD(f_k) = \sqrt{\frac{P(f_k)}{B}} \]  \hspace{1cm} (A.11)
where $B$ is the bandwidth, defined as

$$B = \frac{f_{\text{sample}}}{N} \quad (A.12)$$

If we combine this information with equations A.8-A.10, we get

$$PSD(0) = \sqrt{\frac{1}{f_{\text{sample}}N} |Y(0)|^2} \quad (A.13)$$

$$PSD(f_k) = \sqrt{\frac{2}{f_{\text{sample}}N} |Y(f_k)|^2} \quad k = 1, \ldots, \left(\frac{N}{2} - 1\right) \quad (A.14)$$

$$PSD(f_{N/2}) = \sqrt{\frac{1}{f_{\text{sample}}N} |Y(f_{N/2})|^2} \quad (A.15)$$

### A.4 What is $N_{tt'}^{-1}$?

For a noise timestream chunk of length $n_{TOD}$, we have

The time-time noise autocorrelation is just the inverse FFT of the magnitude squared of the FFT of the noise. If $y(t)$ is a noise timestream and $Y(f)$ is its Fourier transform, then

$$N_{tt'} = \text{FFT}^{-1} \left[ \frac{|Y(f)|^2}{N} \right] \quad (A.16)$$

Where $f$ includes both positive and negative frequencies. Obviously this expression just gives us the first row of the $N_{tt'}$ matrix, which is all we need since it is Toeplitz and circulant. The normalization factor of $1/N$ is necessary so that for the white noise case, $N_{tt'}$ equals the RMS squared for $t = t'$ as expected. Note that this is separate from the standard factor of $1/N$ required when taking the inverse Fourier transform with FFTW.
Since $N_{tt}$ is Toeplitz, we can approximate the first row of the inverse of this matrix by

$$N_{tt}^{-1} \approx \text{FFT}^{-1} \left[ \frac{N}{|Y(f)|^2} \right]$$

(A.17)

### A.5 A Note on RFFTW

When using RFFTW, you only specify the positive frequency values for frequency domain data. This is because the negative frequency values are simply the complex conjugate of the positive frequency values. For example, if our time vector $y(t)$ is Real, then its FFT would be stored in a halfcomplex FFTW vector like this (recall that the zero and Nyquist frequency terms are always real):

$$Y[0] = Y(0) \quad (A.18)$$

$$Y[k] = \Re(Y(f_k)) \quad k = 1, \ldots, \left( \frac{N}{2} - 1 \right) \quad (A.19)$$

$$Y[N - k] = \Im(Y(f_k)) \quad k = 1, \ldots, \left( \frac{N}{2} - 1 \right) \quad (A.20)$$

$$Y[N/2] = Y(f_{N/2}) \quad (A.21)$$

As another example, the square of the FFT (which is real) would be stored in halfcomplex form like this:

$$Y^2[0] = |Y(0)|^2 \quad (A.22)$$

$$Y^2[k] = |Y(f_k)|^2 \quad k = 1, \ldots, \left( \frac{N}{2} - 1 \right) \quad (A.23)$$

$$Y^2[N - k] = 0 \quad k = 1, \ldots, \left( \frac{N}{2} - 1 \right) \quad (A.24)$$

$$Y^2[N/2] = |Y(f_{N/2})|^2 \quad (A.25)$$
One other thing to remember when using FFTW is that when taking the inverse FFT, you must divide by $N$.

**A.6 Effects of Windowing and Averaging on the PSD**

Let’s take a look at a more realistic situation. This discussion follows the one in Numerical Recipes (REF). Suppose we have a chunk of $M$ data points for which we want to calculate the Power Spectrum, PSD, and $N_{tt}^{-1}$. First, let’s assume that we can divide the chunk into $L$ overlapping segments of $N$ points where each segment overlaps its neighbors by half its length. Here is a graphical example with $L = 19$ and $M = 10N$:

One way to reduce the noise in the power spectrum is to compute the power spectrum in each segment and then average the results together:

$$P(f_k) = \frac{1}{L} \sum_{i=0}^{L-1} P_i(f_k)$$  \hspace{1cm} (A.26)

Another important technique for improving the estimate of the power spectrum is to window the data (multiply it) by a symmetric function that is peaked at the center of
the segment and tails off at either end. For example the Welch window looks like this:

\[
    w(x) = 1 - \left( \frac{x - \frac{N}{2}}{\frac{N}{2}} \right)^2 \quad x = 0, \ldots, (N - 1) \tag{A.27}
\]

For each segment, we multiply the \( N \) timestream samples by the window function. Let’s define the windowed data as \( d \) and the corresponding FFT as \( D \).

\[
    D(f) = \text{FFT} [d(t)] = \text{FFT} [w(t)y(t)] \tag{A.28}
\]

In terms of the windowed data, the power spectrum is now written as

\[
    P(0) = \frac{1}{W} |D(0)|^2 \tag{A.29}
\]

\[
    P(f_k) = \frac{2}{W} |D(f_k)|^2 \quad k = 1, \ldots, \left( \frac{N}{2} - 1 \right) \tag{A.30}
\]

\[
    P(f_{N/2}) = \frac{1}{W} |D(f_{N/2})|^2 \tag{A.31}
\]

where \( W \) is the sum squared window:

\[
    W = N \sum_{x=0}^{N-1} [w(x)]^2 \tag{A.32}
\]

So if we window each segment, compute the power spectrum, and average these spectra together, then we end up with

\[
    P(0) = \frac{1}{W \cdot L} \sum_{i=0}^{L-1} |D_i(0)|^2 \tag{A.33}
\]

\[
    P(f_k) = \frac{2}{W \cdot L} \sum_{i=0}^{L-1} |D_i(f_k)|^2 \quad k = 1, \ldots, \left( \frac{N}{2} - 1 \right) \tag{A.34}
\]

\[
    P(f_{N/2}) = \frac{1}{W \cdot L} \sum_{i=0}^{L-1} |D_i(f_{N/2})|^2 \tag{A.35}
\]
If we divide this expression by the bandwidth to get the PSD, we end up with

\[
\text{PSD}(0) = \frac{1}{\sqrt{f_{\text{sample}} L}} \left( \sum_{x=0}^{N-1} |w(x)|^2 \right)^{L-1} \sum_{i=0}^{L-1} |D_i(0)|^2
\]

\[
\text{PSD}(f_k) = \frac{2}{\sqrt{f_{\text{sample}} L}} \left( \sum_{x=0}^{N-1} |w(x)|^2 \right)^{L-1} \sum_{i=0}^{L-1} |D_i(f_k)|^2 \quad k = 1, \ldots, \left( \frac{N}{2} \right)
\]

\[
\text{PSD}(f_{N/2}) = \frac{1}{\sqrt{f_{\text{sample}} L}} \left( \sum_{x=0}^{N-1} |w(x)|^2 \right)^{L-1} \sum_{i=0}^{L-1} |D_i(f_{N/2})|^2
\]

**A.7 From PSD to Unconstrained Noise Realization**

When creating fake noise timestreams, it is necessary to use the noise PSDs estimated from the real data. Here is the step-by-step approach I use to do this:

1. For white noise, the normalization criterion is simply related to the width of the Gaussian distribution by

\[
\sum_{k=0}^{N/2+1} P(f_k) = \frac{1}{N} \sum_{j=0}^{N-1} |y(t_j)|^2 = \sigma^2
\]

This can be proven explicitly by integrating the function

\[
\sum_{j=0}^{N-1} |y(t_j)|^2 = \frac{2}{\sigma \sqrt{2\pi}} \int_0^\infty dy \ (y)^2 \exp \left( -\frac{y^2}{2\sigma^2} \right)
\]

Using this, we can find the sum of the power spectrum from the PSD and then generate a white noise timestream with the proper \( \sigma \) (by drawing random values from a Gaussian distribution).

\[
\sum_{k=0}^{N/2+1} P(f_k) = \sum_{k=0}^{N/2+1} B |\text{PSD}(f_k)|^2 = \sigma^2
\]
2. In order to properly weight the frequency components of the timestream, first take the FFT of the white noise timestream. For each frequency, compute the magnitude and phase of the complex Fourier component. Keep the phase value, but set the magnitude so that it agrees with the known power spectrum.

\[
\text{Magnitude} = \sqrt{N^2 P(f)} = \sqrt{N^2 B [PSD(f_k)]^2}
\] (A.42)

3. For each frequency, recompute the real and imaginary parts from the magnitude and phase. Then take the inverse FFT to obtain a noise timestream with the desired spectral properties.
Appendix B

Covariance Matrices

When dealing with pixel domain signal and noise covariance matrices (and the data covariance constructed from their sum), we are often faced with matrices which are poorly conditioned. In the case of the noise covariance, this is due to the combination of scan strategy and time domain noise properties producing a pixel domain noise covariance which has essentially infinite noise in some spectral modes. For the signal covariance, some combinations of sky pixel locations do not effectively constrain some spectral modes.

In order to get around these problems, a general solution was developed. We first carry out all of the iterative likelihood calculations in a basis where the noise covariance is transformed into the identity matrix. This signal-to-noise basis or Karhunen-Loeve transform[6] results in a data covariance whose large eigenmodes are the physically important ones (the ones with high S/N). When inverting the data covariance, we then
marginalize over (fix at zero) all modes whose eigenvalues are below some threshold based on the maximum desired condition number of the matrix. The result of this process is not a true matrix inverse, but all of the physically important modes remain intact.

The eigendecomposition is carried out using the PDSYEV routine from the SCALA-PACK library. For matrices of the size discussed here (33000 x 33000), the inversion using this procedure requires a few minutes on 1024 2.6GHz Opteron cores.